Chapter 3

Credit and Business Cycles

Here I present a model of the interaction between credit and business cycles. In representative agent models, remember, no lending actually takes place!

The literature on the topic has emphasized the idea that a credit market imperfections can magnify the effects on the real economy of given technology or monetary shocks. This happens because a positive shock increases income of owners of production technology; this rise in net worth lowers the costs associated with external financing of investment projects, allowing for increased investment. This serves to amplify and propagate the effects of a shock through time...

3.1 The modelling choices

We now develop a model that illustrates the role of debt, net worth and asset price fluctuations on equilibrium output. You can consider this model as an extension of the real business cycle model which allows for financial factors to play a role in business fluctuations.

The model has to be as simple as possible. In fact it can be made quite simple even though it features heterogenous agents by adding to it the following features:

1. a constant interest rate (one less variable)

2. no labor supply decision

3. no capital accumulation

4. only one asset that can be used for production

3.2 A basic model

The reference is the paper "Credit Cycles" by Kiyotaki and Moore (1997), JPE, 211-247, available on JSTOR. We use a slightly modified version.

There are two types of agents, farmers (productive) and gatherers (unproductive). They both have linear preferences over consumption, but they discount the future differently. Farmers have a discount factor of γ, gatherers have a discount factor of β, where $\beta > \gamma$. They produce a final good y_t using land h_t.

The production functions are respectively described by:

$$\begin{aligned} y_t &= A_t h_{t-1}^{\nu} \\ y'_t &= A'_t h'^{\mu}_{t-1} \end{aligned}$$

3.2.1 Gatherers

The gatherers will be the unproductive agents, who are not constrained, solve the following problems.

$$\max_{h'_t, b'_t} \; E_0 \left(\sum_{t=0}^{\infty} \beta^t c'_t \right)$$
$$\text{s.t } c'_t + q_t h'_t + R b'_{t-1} = y'_t + q_t h'_{t-1} + b'_t$$

yields:

$$\begin{aligned} q_t &= \beta E_t \left(q_{t+1} + \mu \frac{y'_{t+1}}{h'_t} \right) \\ \beta R &= 1 \end{aligned}$$

3.2.2 Farmers

Farmers maximize

$$\max_{b_t, h_t} E_0 \left(\sum_{t=0}^{\infty} \gamma^t c_t \right)$$

their flows of funds is

$$c_t + q_t h_t + R b_{t-1} = y_t + q_t h_{t-1} + b_t$$

where on the RHS we have the sources of fund, on the LHS we have its uses. The amount of claims farmers can issue is bound by:

$$R b_t \leq m E_t \left(q_{t+1} h_t \right)$$

Remark 6 *Why can't the farmers borrow more than the $mE_t\left(q_{t+1}h_t/R\right)$? G lends some goods to F, who in turn promises him to pay him back at some future date. The assumption here is that the farmer's labor input is critical for production: once the farmer starts producing, no one can replace her, and the farmer cannot commit to repay his debt. If the creditor* *tries to extract too much from the farmer, the farmer can simply walk away from the land. Current production is lost, and the farmer only recovers the value of the land. This in turn limits her ability to borrow.*

The Lagrangean for this problem is:

$$\max_{b_t, h_t} E_t \left(c_t + \gamma c_{t+1} + ... - \lambda_t \left(R b_t - m q_{t+1} h_t \right) - ... \right)$$

for each t. The first order conditions are choosing b_t and h_t respectively

$$\begin{aligned} 1 &= \gamma R + \lambda_t R \\ q_t &= \gamma E_t \left(q_{t+1} + \nu \frac{y_{t+1}}{h_t} \right) + E_t \left(\lambda_t m q_{t+1} \right) \end{aligned}$$

3.2.3 Equilibrium

How does the equilibrium look like? Again, remember that there are the bonds, goods and asset market, together with the first order conditions.

From the Euler equation for gatherers, we know that:

$$\beta R = 1$$

coupled with that of farmers

$$\lambda_t = \lambda = \frac{1 - \gamma R}{R} = \beta - \gamma > 0$$

which implies that the borrowing constraint will be binding near the ss.

Once we collect all of them, we obtain the following equations:

$$c'_t + q_t h'_t + R b'_{t-1} = y'_t + q_t h'_{t-1} + b'_t \quad (1)$$

$$c_t + q_t h_t + R b_{t-1} = y_t + q_t h_{t-1} + b_t \quad (2)$$

$$q_t = \left(m\beta + (1-m)\gamma \right) E_t q_{t+1} + \gamma v E_t \left(\frac{y_{t+1}}{h_t} \right) \quad (3)$$

$$q_t = \beta E_t q_{t+1} + \beta \mu E_t \left(\frac{y'_{t+1}}{h'_t} \right) \quad (4)$$

$$R b_t = m E_t \left(q_{t+1} h_t \right) \quad (5)$$

To this animal, we need to add market clearing conditions. Normalize asset supply to 1.

$$h_t + h'_t = 1 \quad (6)$$

$$b_t + b'_t = 0 \quad (7)$$

$$y_t = A_t h_{t-1}^{\nu} \quad (8)$$

$$y'_t = A'_t h'^{\mu}_{t-1} \quad (9)$$

Remember that if the bond market clears, so will the goods one, so we will not need to write down the latter.

A quick check: 9 equations, whereas the variables are $c'_t \quad c_t \quad y_t \quad y'_t \quad h_t \quad h'_t \quad b_t \quad b'_t \quad q_t$. Great!!!

3.2.4 Simplify

Most of the equations are trivial. Also notice that c'_t and c_t only appear in 1 and 2, so they will adjust so that 1 and 2 hold.

Similarly, b_t will be given by equation 5, so we solve it by hand as well.

To simplify matters, Kiyotaki and Moore also assume that $\nu = 1$, so that the production technology of the farmers is linear. This implies that all the dynamics of the system can be analyzed with reference to the following guys (define $\phi = m\beta + (1-m)\gamma$; you can think of ϕ as an average of the two discount factors, the higher m, the higher ϕ)

$$q_t = \phi E_t q_{t+1} + \gamma E_t A_{t+1} \tag{3}$$

$$q_t = \beta E_t q_{t+1} + \beta\mu E_t \left(\frac{A'_{t+1}}{(1-h_t)^{1-\mu}} \right) \tag{4}$$

3.2.5 Steady state

Let us look at the steady state first. From 3

$$q = \frac{\gamma A}{1-\phi}$$

from 4

$$q(1-\beta) = \frac{\beta\mu A'}{(1-h)^{1-\mu}}$$

which gives us h.

$$1-h = \left(\frac{\beta\mu A'(1-\phi)}{\gamma A(1-\beta)} \right)^{\frac{1}{1-\mu}}$$

Remark 7 *Crucial. What is MPK in the farmer sector? Simply, it is A*

Remark 8 *What is MPK in the other sector? Simply from the optimality conditions of gatherers:*

$$q = \beta q + \beta MPK$$

$$MPK = \frac{q(1-\beta)}{\beta} = \frac{\gamma(1-\beta)}{\beta(1-\phi)} A$$

which can be solved for q. It turns out that given $\gamma < \phi < \beta$, then $\frac{\gamma(1-\beta)}{\beta(1-\phi)} < 1$ so that the MPK in the gatherers sector is below that of the farming sector. This implies that in equilibrium the allocation of land is inefficient since its marginal product is not equated across the two sectors.

From 5 we can also derive:

$$b = \beta mqh$$

as for the other variables

$$c = y - (R-1)b = Ah - (1-\beta)mqh = A\left(1 - \frac{m\gamma(1-\beta)}{1-\phi}\right)h$$

$$c' = y' + (R-1)b$$

3.2.6 Log-linearisations

To consider the effects on the economy of productivity shocks, we linearize around the steady state and calculate the effect of a 1% increase in productivity.

Kiyotaki and Moore consider an increase in productivity for both agents, that is $\widehat{A}_t = \widehat{A}'_t$. Let us also assume that once a technology shock occurs, it follows an AR(1) process of the form:

$$\widehat{A}_t = \rho \widehat{A}_{t-1} + \widehat{e}_t^A$$

From equation 3 we obtain:

$$\begin{aligned} q_t &= \phi E_t q_{t+1} + \gamma E_t A_{t+1} \\ \widehat{q}_t &= \phi E_t \widehat{q}_{t+1} + (1-\phi) E_t \widehat{A}_{t+1} \end{aligned} \tag{L1}$$

which can be solved forward to obtain:

$$\widehat{q}_t = \frac{1-\phi}{1-\phi\rho} E_t \widehat{A}_{t+1} = \frac{1-\phi}{1-\phi\rho} \rho \widehat{A}_t \tag{dr1}$$

where ρ is the persistence of the technology shock.

For equation 4:
$\beta\mu \frac{A}{(1-h)^{1-\mu}} h = q(1-\beta)$

$$\begin{aligned} q_t &= \beta E_t q_{t+1} + \beta\mu E_t \left(\frac{A'_{t+1}}{(1-h_t)^{1-\mu}} \right) \\ dq_t &= \beta E_t (dq_{t+1}) + \underbrace{\beta\mu \frac{A'}{(1-h)^{1-\mu}}}_{q(1-\beta)} \frac{dE_t A'_{t+1}}{A'} + \underbrace{\beta\mu \frac{A'}{(1-h)^{1-\mu}}}_{q(1-\beta)} \frac{(1-\mu) h}{(1-h)} \frac{dh_t}{h} \\ & \text{divide everything by } q \\ \widehat{q}_t &= \beta E_t \widehat{q}_{t+1} + (1-\beta) \left(E_t \widehat{A}_{t+1} + \frac{1}{\eta} \widehat{h}_t \right) \end{aligned} \tag{L2}$$

where $\eta = \frac{1-h}{(1-\mu)h}$ is the inverse elasticity of asset demand from the farmers with respect to the price.

L1 and L2 together yield:

$$\frac{1-\phi}{1-\phi\rho} \rho \widehat{A}_t = \frac{1-\phi}{1-\phi\rho} \beta \rho^2 \widehat{A}_t + (1-\beta) \left(\rho \widehat{A}_t + \frac{1}{\eta} \widehat{h}_t \right)$$

which can be solved for the response of $\widehat{h}_t$ to a productivity shock, that is:

$$\widehat{h}_t = \frac{(1-\phi)(1-\beta\rho) - (1-\phi\rho)(1-\beta)}{(1-\phi\rho)(1-\beta)} \eta \rho \widehat{A}_t \tag{dr2}$$

$(dr1)$ and $(dr2)$ are the two linearized decision rules of our problem. $(dr2)$ shows that h rises with A so long as $0 < \rho < 1$ (the numerator is positive because $\phi < \beta$).

Hence in this problem a productivity shocks raises asset prices which in turn raises asset demand from the most productive agents, who can then produce more and accumulate more: the effects on aggregate activity are therefore amplified by the asset price effect.

In the homework, you are asked to show how aggregate output rises more than the increase in productivity: that is, you are asked to show that:

$$\widehat{Y}_t = \frac{y}{y+y'}\widehat{y}_t + \frac{y'}{y+y'}\widehat{y}'_t > \widehat{A}_t$$

3.3 Forward-lookingness and history dependence

The model here displays forward-lookingness; an increase in asset prices implies that the creditor will be able to recover more from selling the asset whenever the debtor defaults, therefore for each asset price increase he is willing to supply more credit. As credit increases, asset prices rise, aggregate investment rises (since the marginal productivity of debtors' is higher than creditors') and so on and so forth, in a cumulative fashion.

However, it does not display history-dependence. Kiyotaki and Moore say when a shock hits, it is too late to renegotiate, therefore repayments are the same, but current borrowing increases. If consumption is bound by some upper limit, all the new borrowing goes into investment, and this strongly magnifies the response of output to a given productivity shock. The homework questions asks you to analyze this in more detail.