Chapter 4

The dynamic new-keynesian model

Recent years have seen an explosion of models in which there are nominal rigidities; these models have nested the RBC model as a special case.

At least since Keynes, it has been thought that in order to have real effects from monetary actions, it is key to have some degree of nominal rigidity. The question we want to explore is: can a model based on microfoundations based on these features describe some important features of the link between monetary actions and the business cycle?

What do we need in order to get nominal rigidities in the traditional, dynamic general equilibrium model? Well, we need some form of pricing power, for instance coming from monopolistic competition, and therefore some heterogeneity among goods.

The main actors of the DNK model are:

agents/mkts	final good	intermediate	labor	profit	money	bonds	
Household	$-Pc$		WL	PF	$M_{-1} - M + PT$	$RB_{-1} - B$	$= 0$
final firm	PY	$-\int_0^1 P_j Y_j dj$					$= 0$
interm. firms		$\int_0^1 P_j Y_j dj$	$-WL$	$-PF$			$= 0$
Govt					$M - M_{-1} - PT$		$= 0$
equilibrium	$= 0$	$= 0$	$= 0$	$= 0$	$= 0$	$= 0$	

- households: make consumption and labor supply decisions, demand money and bonds
- final good firms: produce final goods Y_t from intermediate goods Y_{jt}
- intermediate good firm: use labor to produce intermediate goods Y_{jt}. Over each of this goods they have monopoly power. Demand labor. Can set price of good Y_j
- government: runs monetary policy.

4.1 Households

There is a continuum of infinitely-lived individuals, whose total is normalized to 1. They choose consumption c_t, labor L_t, money M_t and bonds B_t in order to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\rho}}{1-\rho} - \frac{1}{\eta} (L_t)^\eta + \chi \ln \frac{M_t}{P_t} \right)$$

where E_0 denotes the expectation operator conditional on time 0 information, β is the discount factor, subject to:

$$c_t + \frac{B_t}{P_t} = \frac{R_{t-1}}{P_t} B_{t-1} + \frac{W_t}{P_t} L_t + F_t - \frac{M_t - M_{t-1}}{P_t} + T_t$$

where the wage is $w_t \equiv W_t/P_t$, and bonds pay the predetermined nominal interest rate R_{t-1}; F_t denotes lump-sum dividends received from ownership of intermediate goods firms (whose problems are described below); the last three terms indicate net transfers from the central bank that are financed by printing money.

Let $\Pi_t \equiv P_t/P_{t-1}$ denote the gross rate of inflation. Solving this problem yields first order conditions for consumption/saving, labour supply and money demand:

$$\frac{1}{c_t^\rho} = \beta E_t \left(\frac{R_t}{\Pi_{t+1} c_{t+1}^\rho} \right) \qquad \text{(Euler)}$$

$$\frac{w_t}{c_t^\rho} = L_t^{\eta-1} \qquad \text{(LS)}$$

$$\frac{1}{c_t^\rho} = E_t \left(\beta \frac{1}{\Pi_{t+1}} \frac{1}{c_{t+1}^\rho} \right) + \frac{\chi}{m_t} \qquad \text{(MD)}$$

where m_t are real balances (M_t/P_t).

4.2 Final-goods firm

There is a final-goods sector where a representative firm produces the final good Y_t using intermediate goods Y_{jt}. Total final goods are given by the CES aggregator of the different quantities of intermediate goods produced:

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon > 1$.

The firm buys inputs Y_{jt} and produces the final good in order to maximize profits, taking P_{jt} as given:

$$\max_{Y_{jt}} Y_t - \frac{1}{P_t} \int_0^1 P_{jt} Y_{jt} dj$$

where P_t is an index (to be determined) that converts nominal expenditures into real expenditures.

Optimal choice of Y_{jt} solves:

$$\frac{\partial}{\partial Y_{jt}}\left[\left(\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}}dj\right)^{\frac{\varepsilon}{\varepsilon-1}} - \frac{1}{P_t}\int_0^1 P_{jt}Y_{jt}dj\right] = 0$$

$$\Leftrightarrow \frac{\varepsilon}{\varepsilon-1}\underbrace{\left(\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}}dj\right)^{\frac{1}{\varepsilon-1}}}_{Y_t^{1/\varepsilon}}\frac{\varepsilon-1}{\varepsilon}Y_{jt}^{\frac{-1}{\varepsilon}} = \frac{P_{jt}}{P_t}$$

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} Y_t$$

From $Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} Y_t$ (demand for each input) use CES to obtain:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon}\left(\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}}dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

and solving for P_t

$$P_t = \frac{P_{jt}\left(\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}}dj\right)^{\frac{1}{1-\varepsilon}}}{Y_{jt}^{-\frac{1}{\varepsilon}}} = \left(\int_0^1 P_{jt}^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$$

P_t represents the minimum cost of achieving one unit of the final-goods bundle Y_t. For this reason we interpret P_t as the aggregate price index.

In equilibrium profits in this sector will be equal to zero: this occurs because the production function in the final goods firm problem has constant returns to scale, therefore from the Euler's theorem there cannot be profits.

4.3 Intermediate goods

The intermediate goods sector is made by a continuum of monopolistically competitive firms owned by consumers, indexed by $j \in (0, 1)$.

4.3.1 The constraints

Each firm, as we saw above, faces a downward sloping demand for its product. It uses labor to produce output according to the following technology:

$$Y_{jt} = A_t L_{jt}$$

Each producer chooses her own sale P_{jt} taking as given the demand curve. He can reset his price only when given the chance of doing so, which occurs with probability $1-\theta$ in every period.

So, how many constraints do intermediate goods firms face?

1. the production constraint: $Y_{jt} = A_t L_{jt}$
2. the demand curve $Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} Y_t$
3. the fact that prices can be adjusted only with probability $1-\theta$. We follow Calvo (1983) and assume that every period only a random fraction of firms is setting prices. Each period, this fraction is independent from the previous period.

We can break this problem down into two sub-problems. As a cost minimizer and as a price setter.

4.3.2 Producer as a cost minimizer

Consider the cost minimization problem first, conditional on the output Y_{jt} produced. This problem involves minimizing $W_t L_{jt}$ subject to producing $Y_{jt} = A_t L_{jt}$ (there is no sub-index on W_t since all sectors where labor is employed must pay same wage in equilibrium). In real terms this problem can be written as

$$\min_{L_{jt}} \frac{W_t}{P_t} L_{jt} + Z_t \left(Y_{jt} - A_t L_{jt}\right) \qquad [Z_t]$$

where Z_t is multiplier associated with the constraint. The first order condition implies:

$$\frac{Y_{jt}}{L_{jt}} = \frac{1}{Z_t}\frac{W_t}{P_t} \equiv X_t \frac{W_t}{P_t} \tag{LD}$$

notice that this first order condition suggests than we can write the real cost function as

$$COST_t = \frac{W_t}{P_t} L_{jt} = Z_t Y_{jt}$$

For this reason, we can think of Z_t as real marginal cost; we can likewise define its inverse $X_t = 1/Z_t$ as the markup. Given cost minimization, the firm takes Z_t as given, when choosing the output price, to which we turn now.

4.3.3 Producer as a price setter

4.3.3.1 Digression: the problem with flexible prices (and the macro equilibrium with flexible prices)

In order to warm yourself up, consider the problem of a monopolistic producer who has the chance to change her prices every period. The cost minimization problem is the same as before. On the revenue side, define $r_{jt} \equiv P_{jt}/P_t$ the relative price that the producer charges. The maximization problem will be:

$$\max_{r_{jt}} r_{jt} Y_{jt} - Z_t Y_{jt}$$

where $Y_{jt} = r_{jt}^{-\varepsilon} Y_t$. Optimal choice of r_{jt} will imply

$$Y_{jt}^* + r_{jt}^* \frac{\partial Y_{jt}^*}{\partial r_{jt}^*} - Z_t \frac{\partial Y_{jt}^*}{\partial r_{jt}^*} = 0$$

$$Y_{jt}^* \left(1 + \frac{r_{jt}^*}{Y_{jt}^*} \frac{\partial Y_{jt}^*}{\partial r_{jt}^*} - \frac{Z_t}{r_{jt}^*} \frac{r_{jt}^*}{Y_{jt}^*} \frac{\partial Y_{jt}^*}{\partial r_{jt}^*}\right) = 0$$

$$r_{jt}^* = \frac{P_{jt}^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} Z_t$$

that is, the relative price would be a constant markup $X \equiv \frac{\varepsilon}{\varepsilon-1}$ over the real marginal cost.

Remark 9 *This condition is crucial because with monopolistic competition but flexible prices we would derive a neutrality result similar to that of Sidrauski model. In the symmetric equilibrium, $P_{jt} = P_t$, hence $Z_t = \frac{\varepsilon-1}{\varepsilon} = \frac{1}{X} < 1$ for all t. Combining labor supply (equation LS) and labor demand (equation LD) and imposing market clearing $Y_t = C_t$ would give*

$$w_t = \underbrace{Y_t^{\rho} L_t^{\eta-1}}_{\text{labor supply}} = \underbrace{A_t / X}_{\text{labor demand}}$$

using $L_t = Y_t / A_t$ in the symmetric equilibrium we will have:

$$Y_t = \left(\frac{A_t^{\eta}}{X}\right)^{\frac{1}{\rho+\eta-1}}$$

so that output in the model is a function only of technology. In static terms, output would be suboptimally low.

4.3.3.2 The problem with sticky prices

To begin with, at any point in time if some intermediate good producers can change prices and others cannot, the average price level will be a CES aggregate of all prices in the economy, and will be

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta) \left(P_t^*\right)^{1-\varepsilon} \tag{*}$$

where P_{t-1} is previous price level, and P_t^* is avg price level chosen by those who have the chance to change prices. It is at these guys that we look now.

Consider the intermediate goods producer who has a chance $1 - \theta$ to reset prices at time t. Call P^* the reset price. The demand curve is:

$$Y_{jt+k}^* = \left(P_{jt}^* / P_{t+k}\right)^{-\varepsilon} Y_{t+k}$$

for any period $k \geq 0$ for which he will keep that price.

His maximization problem is:

$$\max_{P_{jt}^*} \sum_{k=0}^{\infty} (\theta\beta)^k E_t \left(\Lambda_{t,k} \left(\frac{P_{jt}^*}{P_{t+k}} - Z_{t+k}\right) Y_{jt+k}^*\right) \tag{\#}$$

$$\Lambda_{t,k} = \left(C_t / C_{t+k}\right)^{\rho}$$

where Z_t is the real marginal cost. θ represents the probability that the price P_j^* chosen at t will still apply in later periods. This expression is the "expected discounted sum of all profits that the price setter will make conditional on his choice of P_{jt}^* and weighted by how likely P_{jt}^* is to stay in place in future periods".

At time t, the price setter chooses P^* to maximize profit. Differentiate # with respect to $P_{jt}^*/P_{t+k} \equiv r_{jt}^*$ (the relative price) to obtain

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left[\Lambda_{t,k} \left(Y_{jt+k}^* + r_{jt}^* \frac{\partial Y_{jt+k}^*}{\partial r_{jt}^*} - Z_{t+k} \frac{\partial Y_{jt+k}^*}{\partial r_{jt}^*} \right) \right] = 0$$

take Y_{jt+k}^* out, isolate elasticity of Y_{jt+k}^* wrt r_{jt}^*

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left[\Lambda_{t,k} Y_{jt+k}^* \left(1 + \frac{r_{jt}^*}{Y_{jt+k}^*} \frac{\partial Y_{jt+k}^*}{\partial r_{jt}^*} - \frac{Z_{t+k}}{r_{jt}^*} \frac{r_{jt}^*}{Y_{jt+k}^*} \frac{\partial Y_{jt+k}^*}{\partial r_{jt}^*} \right) \right] = 0$$

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left[\Lambda_{t,k} Y_{jt+k}^* \left(1 - \varepsilon + \frac{Z_{t+k}}{r_{jt}^*} \varepsilon \right) \right] = 0$$

multiply inside brackets by $\dfrac{r_{jt}^*}{1-\varepsilon}$

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left[\Lambda_{t,k} Y_{jt+k}^* \left(\frac{P_{jt}^*}{P_{t+k}} - \frac{\varepsilon}{\varepsilon - 1} Z_{t+k} \right) \right] = 0$$

In equilibrium, all the firms that reset the price choose the same price (and face the same demand), hence

$$P_{jt}^* = P_t^*$$

These two expressions enter the equilibrium (using $X = \frac{\varepsilon}{\varepsilon-1}$ = steady state markup). One is $*$ that we derived above, the other is:

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left[\Lambda_{t,k} Y_{t+k}^* \left(\frac{P_t^*}{P_{t+k}} - X \frac{Z_{t+k}^n}{P_{t+k}} \right) \right] = 0 \qquad (**)$$

where $Z_{t+k} = Z_{t+k}^n / P_{t+k}$. Rearrange the expression above to obtain:

$$P_t^* = X \frac{\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left[\Lambda_{t,k} Y_{t+k}^* P_{t+k}^{-1} Z_{t+k}^n \right]}{\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left[\Lambda_{t,k} Y_{t+k}^* P_{t+k}^{-1} \right]} = X \sum_{k=0}^{\infty} \phi_{t,k} Z_{t+k}^n$$

where $\phi_{t,k} = \frac{(\theta\beta)^k E_t \left[\Lambda_{t,k} Y_{t+k}^* P_{t+k}^{-1} \right]}{\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left[\Lambda_{t,k} Y_{t+k}^* P_{t+k}^{-1} \right]}$. This expression says that the optimal price is a weighted average of current and expected future nominal marginal costs. Weigths depend on expected demand in the future, and how quickly firm discounts profits.

Therefore you can notice the following:

- Under purely flexible prices, $\theta = 0$: the markup is a constant. $P_t^* = X Z_t^n$ and optimal prices are a multiple X of the marginal cost.

- When $\theta > 0$, the optimal price depends on future expected values of aggregate variables (P_{t+k}, Y_{t+k}) as well as future nominal marginal costs Z^n_{t+k}. Put differently, one can see that all the fluctuations in the markup are due to firms being unable to adjust prices.

4.4 The equilibrium

4.4.1 Closing the model

We need to combine everything, impose market clearing, and linearize around the steady state.

Total output in economy is:

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} = \left[\int_0^1 (A_t L_{jt})^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

It is not possible to simplify this expression since input usages across firms differs. However the linear aggregator $Y_t' = \int_0^1 Y_{jt} dj$ is approximately equal to Y_t within a local region of the steady state. Hence for local analysis we can simply use

$$Y_t = A_t L_t$$

Goods market clearing is simply $Y_t = C_t$. Trivially, bond market clearing implies $B_t = 0$.

4.4.2 Monetary policy

We assume that the central bank policy sets the dynamics of money supply in a way to achieve a target level of the interest rate. This way, the money demand equation becomes redundant since it only serves to determine the behavior of endogenous money.

To better gain insight into this, consider money demand:

$$\frac{M_t}{P_t} = \chi c_t^{\rho} \frac{R_t}{R_t - 1}$$

In log-linear terms this becomes:

$$\widehat{R}_t = \psi \left(\rho \widehat{C}_t - \widehat{m}_t\right) \tag{md}$$

where ψ is some positive coefficient. So far, we have specified central bank policy as control over a monetary aggregate, for instance, for constant money supply:

$$\widehat{M}_t = 0$$

However, one can think of several other monetary rules: for instance, if the central banks wants to peg the interest rate, it simply pegs nominal consumption growth so that:

$$\widehat{M}_t = \widehat{P}_t + \rho \widehat{C}_t \tag{ms}$$

but this implies that we can rewrite money market equilibrium as:

$$\widehat{R}_t = 0$$

and this is an interest rate rule. The point I want to make is: if money enters separably the utility function, we can forget about money demand in the model, and we can close the model by specifying any process for the policy instrument (M or R) we like to consider.

4.4.3 The equilibrium in levels

Let us look at the equation summarising the model:

$$\frac{1}{Y_t^\rho} = \beta E_t\left(\frac{R_t P_t}{P_{t+1}Y_{t+1}^\rho}\right) \quad (1)$$

$$Y_t^{1-\rho-\eta} = A_t^{-\eta}/Z_t \quad (2)$$

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta)\, E_t\left(X\sum\nolimits_{k=0}^{\infty}\phi_{t,k}Z_{t+k}P_{t+k}\right)^{1-\varepsilon} \quad (3)$$

$$R_t = R_{t-1}^{\phi_r}\left(\left(\frac{P_t}{P_{t-1}}\right)^{1+\phi_\pi} Z_t^{\phi_z}\right)^{1-\phi_r}\varepsilon_{r,t} \quad (4)$$

1. is the aggregate demand equation: it combines goods market clearing with the Euler equation for bonds

2. is the equilibrium in the labor market. Take labour demand (LD) and labor supply (LS) and impose market clearing. Then equate (LD) and (LS) so as to eliminate of the real wage w from that expression. Whenever you have L, remember to replace it with Y/A.

3. is the equation that describes how the aggregate price level is a weigthed average of (1) previous price level P_{t-1} and (2) reset prices P_t^*, which are in turn a function of future expected marginal costs.

4. The last expression is the monetary policy rule. We assume that the central bank chooses money supply so as to set the nominal interest rate to be a function of previous interest rate, current inflation and current real marginal costs. This is a Taylor rule, from John Taylor of Stanford University, who was the first to notice in a 1993 seminal paper that central banks set the interest rate as a function of inflation and output gap (output gap=deviation of output from its natural rate). The last term $\varepsilon_{r,t}$ represents a monetary policy shock.

4.5 The log-linear equilibrium

4.5.1 Linearizing the Phillips curve

Use $Y^*_{t+k} = \left(P^*_t/P_{t+k}\right)^{-\varepsilon} Y_{t+k}$ and cancel out P^*_t in numerator and denominator to obtain:

$$P^*_t = X\frac{\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left(\left(\Lambda_{t,k}P^{\varepsilon-1}_{t+k}Y_{t+k}\right)P_{t+k}Z_{t+k}\right)}{\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left(\Lambda_{t,k}P^{\varepsilon-1}_{t+k}Y_{t+k}\right)}$$

To gain insight into this expression, it is convenient to loglinearize it. Intuitively, we can see that numerator and denominator only differ up to a multiple given by $P_{t+k}Z_{t+k}$, which in turn multiplies $(\theta\beta)^k(1-\theta\beta)$. Hence we can expect that in log-linearising Y, $P^{\varepsilon-1}$ and Λ will cancel out and disappear. Rearranging and dividing by P_t:

$$\frac{P^*_t}{P_t}\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left[\Lambda_{t,k}Y_{t+k}P^{\varepsilon-1}_{t+k}\right] = \frac{1}{P_t}X\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left[\Lambda_{t,k}Z_{t+k}Y_{t+k}P_{t+k}P^{\varepsilon-1}_{t+k}\right]$$

LHS first

$$\left(\widehat{P}^*_t - \widehat{P}_t\right)\sum_{k=0}^{\infty}(\theta\beta)^k\left[\Lambda Y P^{\varepsilon-1}\right] + \sum_{k=0}^{\infty}(\theta\beta)^k\left[\Lambda Y P^{\varepsilon-1}\right]E_t\left(\widehat{\Lambda}_{t,k} + \widehat{Y}_{t+k} + (\varepsilon-1)\widehat{P}_{t+k}\right)$$

RHS next

$$-P_t\sum_{k=0}^{\infty}(\theta\beta)^k\left[\Lambda Y P^{\varepsilon-1}\right] + \frac{X}{P}\sum_{k=0}^{\infty}(\theta\beta)^k\left[\frac{\Lambda Y P^{\varepsilon}}{X}\right]E_t\left(\widehat{\Lambda}_{t,k} + \widehat{Z}_{t+k} + \widehat{Y}_{t+k} + \varepsilon\widehat{P}_{t+k}\right) =$$

$$-P_t\sum_{k=0}^{\infty}(\theta\beta)^k\left[\Lambda Y P^{\varepsilon-1}\right] + \sum_{k=0}^{\infty}(\theta\beta)^k\left[\Lambda Y P^{\varepsilon-1}\right]E_t\left(\widehat{\Lambda}_{t,k} + \widehat{Z}_{t+k} + \widehat{Y}_{t+k} + \varepsilon\widehat{P}_{t+k}\right)$$

hence:

$$\widehat{P}^*_t\sum_{k=0}^{\infty}(\theta\beta)^k = \sum_{k=0}^{\infty}(\theta\beta)^k E_t\left(\widehat{P}_{t+k} + \widehat{Z}_{t+k}\right)$$
$$\widehat{P}^*_t = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left(\widehat{P}_{t+k} + \widehat{Z}_{t+k}\right) \qquad (@)$$

Equation (@) simply states in log-linear terms that the optimal price has to be equal to a weighted average of current and future marginal costs, weighted by the probability that this price will hold in later periods too. So you assign weight 1 to today, weight $\theta\beta$ to tomorrow, $\theta^2\beta^2$ to the day after tomorrow, and so on. Notice the complete forwardlookingness of this expression, and the fact that these weights need to be normalized (the sum of all of them is $\frac{1}{1-\theta\beta}$, whose inverse premultiplies the summation - weights sum up to one -)

But we know that:

$$\begin{aligned}
\widehat{P}_t - \theta \widehat{P}_{t-1} &= (1-\theta)\,\widehat{P}_t^* \\
\widehat{P}_t - \theta \widehat{P}_{t-1} &= (1-\theta)(1-\theta\beta)\,E_t\left(\left(\widehat{P}_t + \widehat{Z}_t\right) + \theta\beta\left(\widehat{P}_{t+1} + \widehat{Z}_{t+1}\right) + \theta^2\beta^2(...) + ...\right) \\
\widehat{P}_t - \theta \widehat{P}_{t-1} &= (1-\theta)(1-\theta\beta)\left(\widehat{P}_t + \widehat{Z}_t\right) + \theta\beta\underbrace{\left(E_t\widehat{P}_{t+1} - \theta\widehat{P}_t\right)}_{\text{next period value of LHS}} \\
\widehat{P}_t - \widehat{P}_{t-1} &= -(1-\theta)\,\widehat{P}_{t-1} + (1-\theta)(1-\theta\beta)\,\widehat{P}_t + \theta\beta\left(E_t\widehat{P}_{t+1} - \theta\widehat{P}_t\right) + (1-\theta)(1-\theta\beta)\,\widehat{Z}_t \\
\widehat{\pi}_t &= (1-\theta)\,\widehat{\pi}_t + \theta\beta E_t\widehat{\pi}_{t+1} + (1-\theta)(1-\theta\beta)\,\widehat{Z}_t
\end{aligned}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\widehat{Z}_t$$

This equation is nothing else but an "expectations augmented Phillips curve", which states that inflation rises when the real marginal costs rise. It takes a while to derive, but again it is nothing else but an aggregate supply curve for the whole economy. Notice that

$$\begin{aligned}
\partial\left(\frac{\partial\widehat{\pi}_t}{\partial\widehat{Z}_t}\right)/\partial\beta &= -(1-\theta) < 0 \\
\partial\left(\frac{\partial\widehat{\pi}_t}{\partial\widehat{Z}_t}\right)/\partial\theta &< 0
\end{aligned}$$

- the higher β, the higher the weight to future $\widehat{Z}_t$'s, and the lower today's elasticity to current marginal cost

- the higher θ, the higher the chance that I will be stuck with my price for a long period, and the higher the elasticity of $\widehat{P}_t^*$ to $\widehat{Z}_t$. However, few prices will be changed in the aggregate, therefore aggregate inflation will not be sensitive to the marginal cost.

4.5.2 The remaining equations

Equations (1), (2) and (4) are already linear in logs.

We assume that a_t and e_t follows $AR(1)$ processes.

4.5.3 The complete log-linear model

From now on, we denote with lowercase variables deviations of variables from their respective steady states. I now work in terms of the markup rather than the real marginal cost. When we log-linearize the 4 expressions above, what we obtain the following system:

$$\begin{aligned} y_t &= E_t y_{t+1} - \frac{1}{\rho}\left(r_t - E_t \pi_{t+1}\right) \\ y_t &= \frac{1}{\eta+\rho-1} z_t + \frac{\eta}{\eta+\rho-1} a_t \\ \pi_t &= \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} z_t + u_t \\ r_t &= \phi_r r_{t-1} + (1-\phi_r)\left((1+\phi_\pi)\pi_t + \phi_z z_t\right) + e_t \end{aligned}$$

Remark 10 *The linearized equations are in the Matlab file, where we use $x_t = -z_t$ (the real marginal cost Z_t is the inverse of the markup X_t in levels, and $x_t = -z_t$ in logs - see equation LD). dnwk.m and dnwk_go.m simulate this model. It has 7 equations rather than four, but you can forget about three of them. One equation is capital demand if you extend this model to have capital as well (you can set the weight on K to be arbitrarily small in the production function, so that equation does not count); the other says that $Y = C$; another defines λ as the marginal utility of consumption.*

It is sometimes convenient to call y_t^n a new variable that defines the equilibrium level of output (the natural output) that would prevail under completely flexible prices ($\theta = 0$). This way z_t can in fact be eliminated. In fact, If firms were able to adjust prices optimally each period, $z_t = 0$ (since $\frac{(1-\theta)(1-\beta\theta)}{\theta} \Rightarrow \infty$) and we would be able to define the flexible price equilibrium values for real interest rate and output, which we call their "natural" rates:

$$\begin{aligned} r^n r_t^n &= \frac{\rho\eta}{\eta+\rho-1}\left(a_t - E_t a_{t+1}\right) \\ y_t^n &= \frac{\eta}{\eta+\rho-1} a_t \end{aligned}$$

We can then derive an expression for x_t as a function of the gap between flexible price and sticky price equilibrium, that is:

$$x_t = (\eta+\rho-1)\left(y_t^n - y_t\right)$$

Hence x is positive whenever y is below y^n, output is below its natural level. It is for this reason that we sometimes refer to x_t as the "output gap", since x is proportional to the shortfall of output from its natural level. y_t^n is an exogenous variable, since it depends only on technology. With this convention, the dynamic new-keynesian model can be rewritten as:

$$\begin{aligned} y_t &= E_t y_{t+1} - \sigma\left(r_t - E_t \pi_{t+1}\right) && \text{(a)} \\ \pi_t &= \lambda\left(y_t - y_t^n\right) + \beta E_t \pi_{t+1} + u_t && \text{(b)} \\ r_t &= \phi_r r_{t-1} + (1-\phi_r)\left((1+\phi_\pi)\pi_t + \phi_x\left(y_t - y_t^n\right)\right) + e_t && \text{(c)} \end{aligned}$$

where $\lambda = (\eta+\rho-1)\frac{(1-\theta)(1-\beta\theta)}{\theta}$, $\sigma = 1/\rho$. Some authors have also postulated cost push shocks u_t, that push inflation up. Some authors refer to the system made by (1), (2), (3) as the "benchmark" dynamic-new keynesian model.

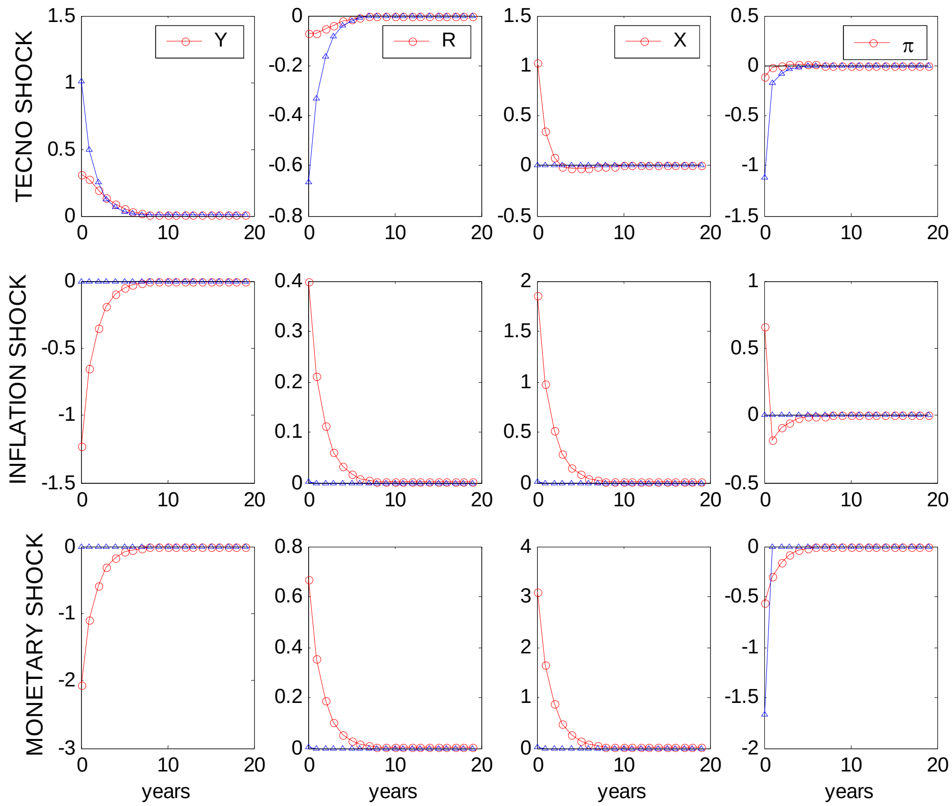

Figure 4.1: Simulations from dnwk.m, flexible (triangles) versus sticky price (circles) model

4.6 The dynamic effects of technology and monetary shocks

The Matlab programs in my webpage will allow you to analyze the dynamics of this model by means of impulse response functions. The plot compares the responses that obtain under flexible prices versus sticky prices. It was generated with $dnwk.m$[1]

Technology

1. Following a rise in technology, marginal costs fall. Since not all prices are free to fall immediately, markups will rise. A fraction $1-\theta$ of "flex price" firms will lower their prices and hire more factors of production. A fraction θ of "fixed price" firms will be unable to lower their prices and to increase their sales and therefore will hire less factors of production. Production rises less than with flexible prices

[1] $\rho_a = 0.5$; $\rho_e = 0.0$; $\rho_u = 0.0$; $\beta = .99$; $\eta = 1.5$; $\theta = .75$ and .0001; $X = 1.1$; $\rho = 1$; $\phi_r = 0.8$; $\phi_\pi = 2$; $\phi_x = 0.0$;

2. The theory of endogenous markup variations provides the crucial link that allows the concerns of RBC models and conventional monetary models to be synthetised. In addition, the markup directly measures the extent to which a condition for efficient resource allocation fails to hold.

Monetary A monetary contraction leads to drop in output, rise in the nominal interest rate, fall in inflation, and a rise in the output gap. These predictions, which are qualitatively in line with the VAR evidence, are hard to obtain in the flexible price model.

Inflation Inflation shocks are important in this setup because they generate a trade-off between output gap versus inflation stabilization.