Technical Appendix
House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle
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The technical appendix to the paper spells out some details about the model and describes in detail the model replication files. In case you have any questions, feel free to email me at `iacoviel@bc.edu`

1. The economic environment

1.1. Entrepreneurs

Entrepreneurs produce wholesale aggregate output, priced at P^w. Output is produced using labor, houses (nominal price of Q_t) and capital. Consumption and investment goods are priced at P_t. Dividing everything by P, using $Q/P = q$ and $P^w/P = 1/X$ and $W/P = w$, the problem can be written as follows:

$$\max_{B_t, I_t, K_t, h_t, L'_t, L''_t} E_0 \sum_{t=0}^{\infty} \gamma^t \ln c_t$$

subject to the following constraints, expressed in real units ($\pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate):

$$Y_t = A_t K_{t-1}^{\mu} h_{t-1}^{\nu} L_t'^{\alpha(1-\mu-\nu)} L_t''^{(1-\alpha)(1-\mu-\nu)} \tag{1}$$

$$\frac{Y_t}{X_t} + b_t = c_t + q_t (h_t - h_{t-1}) + \frac{R_{t-1}}{\pi_t} b_{t-1} + w_t' L_t' + w_t'' L_t'' + I_t + \xi_{K,t} + \xi_{e,t} \tag{2}$$

$$R_t b_t \leq m_t q_{t+1} h_t \pi_{t+1} \tag{3}$$

$$I_t = K_t - (1-\delta) K_{t-1} \tag{4}$$

The adjustment cost functions $\xi_{K,t}$ and $\xi_{e,t}$ are:

$$\xi_{K,t} = \frac{\psi_K}{2\delta} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1}$$

$$\xi_{e,t} = \frac{\psi_H}{2\delta} \left(\frac{h_t - h_{t-1}}{h_{t-1}} \right)^2 h_{t-1}$$

The Lagrangean for the Entrepreneur (omitting the housing adjustment cost, see Section 3.1.2 for details on housing adjustment costs) is:

$$\begin{aligned}
\Lambda &= \log\left(\frac{Y_t}{X_t}+b_t-q_t\left(h_t-h_{t-1}\right)-\frac{R_{t-1}}{\pi_t}b_{t-1}-w_t'L_t'-w_t''L_t''-I_t-\frac{\psi}{2\delta}\left(\frac{I_t}{K_{t-1}}-\delta\right)^2K_{t-1}\right)+ \\
&\gamma\log\left(\frac{Y_{t+1}}{X_{t+1}}+b_{t+1}-q_{t+1}\left(h_{t+1}-h_t\right)-\frac{R_t}{\pi_{t+1}}b_t-w_{t+1}'L_{t+1}'-w_{t+1}''L_{t+1}''-I_{t+1}-\frac{\psi}{2\delta}\left(\frac{I_{t+1}}{K_t}-\delta\right)^2K_t\right) \\
&-\lambda_t\left(R_tb_t-m_tq_{t+1}h_t\pi_{t+1}\right) \\
&-u_t\left(K_t-(1-\delta)K_{t-1}-I_t\right)-\gamma u_{t+1}\left(K_{t+1}-(1-\delta)K_t-I_{t+1}\right)+...
\end{aligned}$$

The first order conditions with respect to B, I, K, h, L', L'' are:

$$\frac{1}{c_t} = E_t\left(\frac{\gamma R_t}{\pi_{t+1}c_{t+1}}\right)+\lambda_tR_t \tag{5}$$

$$u_t = \frac{1}{c_t}\left(1+\frac{\psi}{\delta}\left(\frac{I_t}{K_{t-1}}-\delta\right)\right) \tag{6}$$

$$u_t = \gamma\frac{1}{c_{t+1}}\left(\frac{\psi}{\delta}\left(\frac{I_{t+1}}{K_t}-\delta\right)\frac{I_{t+1}}{K_t}-\frac{\psi}{2\delta}\left(\frac{I_{t+1}}{K_t}-\delta\right)^2\right)+\gamma E_t\left[\frac{\mu Y_{t+1}}{c_{t+1}X_{t+1}K_t}+u_{t+1}(1-\delta)\right] \tag{7}$$

$$\frac{1}{c_t}q_t = E_t\left[\frac{\gamma}{c_{t+1}}\left(\nu\frac{Y_{t+1}}{X_{t+1}h_t}+q_{t+1}\right)+\lambda_tm_t\pi_{t+1}q_{t+1}\right] \tag{8}$$

$$w_t' = \frac{\alpha(1-\mu-\nu)Y_t}{X_tL_t'} \tag{9}$$

$$w_t'' = \frac{(1-\alpha)(1-\mu-\nu)Y_t}{X_tL_t''} \tag{10}$$

The first order conditions 6 and 7 can be combined to yield

$$\frac{1}{c_t}\left(1+\frac{\psi}{\delta}\left(\frac{I_t}{K_{t-1}}-\delta\right)\right)=E_t\left[\frac{\gamma}{c_{t+1}}\left(\frac{\mu Y_{t+1}}{X_{t+1}K_t}+1-\delta+\frac{\psi}{\delta}\left(\frac{I_{t+1}}{K_t}-\delta\right)\left(\frac{1}{2}\left(\frac{I_{t+1}}{K_t}+\delta\right)+1-\delta\right)\right)\right] \tag{11}$$

The formulation of the convex adjustment cost is standard in the literature. See for instance the survey paper by Chirinko (1993). The value of ψ dictates the elasticity of investment to Tobin's u. From linearization of 6

$$\widehat{u}_t+\widehat{c}_t=\psi\left(\widehat{I}_t-\widehat{K}_{t-1}\right)$$

one can see the elasticity of investment to its shadow value is $1/\psi$. Chirinko (1993) surveys the literature based on reduced form estimates of 6.

1.2. Retailers

The retailers' problem is borrowed from Bernanke, Gertler and Gilchrist (1999).

1.3. Unconstrained Households

Households choose consumption c', housing h', money balances M'/P, and labor supply L' in order to maximize:

$$\max_{B_t',h_t',L_t',\frac{M'}{P}} E_0\sum_{t=0}^{\infty}\beta^t\left(\log c_t'+j'\log h_t'-\frac{(L_t')^{\eta'}}{\eta'}+\chi\ln\frac{M_t'}{P_t}\right)$$

where we assume $\beta>\gamma$. Their budget constraint in real terms can be summarised as follows:

$$c_t'+q_t\left(h_t'-h_{t-1}'\right)+\frac{R_{t-1}}{\pi_t}b_{t-1}'=b_t'+w_t'L_t'+F_t\left[-\frac{M_t'-M_{t-1}'}{P_t}+T_t'\right] \tag{12}$$

where F_t denotes dividends received from ownership of retail firms. Solving the problem gives:

$$\frac{1}{c'_t} = E_t\left(\frac{\beta R_t}{\pi_{t+1} c'_{t+1}}\right) \tag{13}$$

$$\frac{q_t}{c'_t} = \frac{j'}{h'_t} + \beta E_t\left(\frac{q_{t+1}}{c'_{t+1}}\right) \tag{14}$$

$$\frac{w'_t}{c'_t} = (L'_t)^{\eta'-1} \tag{15}$$

By taking the first-order condition with respect to money holdings we would obtain a standard money demand equation. Since we focus on monetary rules that target interest rates, so long as utility is separable in money balances, the actual quantity of money has no implications for the rest of the model, and can therefore be ignored.

1.4. Constrained Households

They choose consumption c'', h'', M''/P, and labor supply L'' in order to maximize:

$$\max_{B''_t, h''_t, L''_t, \frac{M''}{P}} E_0 \sum_{t=0}^{\infty} \beta''^t \left(\log c''_t + j'' \log h''_t - \frac{(L''_t)^{\eta''}}{\eta''} + \chi \ln \frac{M''_t}{P_t}\right)$$

where we assume $\beta'' < \beta$. Their budget constraint and borrowing constraints in real terms can be summarised as follows:

$$c''_t + q_t\left(h''_t - h''_{t-1}\right) + \frac{R_{t-1}}{\pi_t} b''_{t-1} = b''_t + w''_t L''_t \left[-\frac{M''_t - M''_{t-1}}{P_t} + T''_t\right] \tag{16}$$

$$R_t b''_t \leq E_t\left(m''_t q_{t+1} h''_t \pi_{t+1}\right) \tag{17}$$

The Lagrangean is:

$$\begin{aligned}
\Lambda = {} & E_t \log\left(b''_t + w''_t L''_t - q_t\left(h''_t - h''_{t-1}\right) - \frac{R_{t-1}}{\pi_t} b''_{t-1} - \frac{M''_t - M''_{t-1}}{P_t} + T''_t\right) - \frac{(L''_t)^{\eta''}}{\eta''} + \\
& \beta'' E_t \log\left(b''_{t+1} + w''_{t+1} L''_{t+1} - q_t\left(h''_{t+1} - h''_t\right) - \frac{R_t}{\pi_{t+1}} b''_t - \frac{M''_{t+1} - M''_t}{P_{t+1}} + T''_{t+1}\right) \\
& + j'' \log h''_t + j'' \beta'' E_t \log h''_{t+1} + .. \\
& - \lambda_t E_t\left(R_t b''_t - m''_t q_{t+1} h''_t \pi_{t+1}\right) - ...
\end{aligned}$$

The first order conditions with respect to respectively B'', h'', L'' are:

$$\frac{1}{c''_t} = E_t\left(\frac{\beta'' R_t}{\pi_{t+1}} \frac{1}{c''_{t+1}}\right) + \lambda''_t R_t \tag{18}$$

$$\frac{q_t}{c''_t} = \frac{j''}{h''_t} + E_t\left(\frac{\beta'' q_{t+1}}{c''_{t+1}} + \lambda''_t m''_t q_{t+1} \pi_{t+1}\right) \tag{19}$$

$$\frac{w''_t}{c''_t} = (L''_t)^{\eta''-1} \tag{20}$$

1.5. Markets and equilibrium

The remaining market clearing equations are given by (ignoring the adjustment costs, which are zero in steady state):

$$
\begin{aligned}
1 &= h_t + h'_t + h''_t && (21) \\
Y_t &= c_t + c'_t + c''_t + I_t && (22) \\
0 &= b_t + b'_t + b''_t && (23)
\end{aligned}
$$

Two of the three flow of funds (the other being satisfied by Walras' law) are given by:

$$
\begin{aligned}
\frac{Y_t}{X_t} + (1-\delta) K_{t-1} - K_t - c_t + b_t &= q_t (h_t - h_{t-1}) + \frac{R_{t-1}}{\pi_t} b_{t-1} + w'_t L'_t + w''_t L''_t && (24) \\
c''_t + q_t \left(h''_t - h''_{t-1}\right) + \frac{R_{t-1}}{\pi_t} b''_{t-1} &= b''_t + w''_t L''_t && (25)
\end{aligned}
$$

In equilibrium, profits of retailers (which are rebated lump-sum to unconstrained households and have no effect on the dynamics) will equal $F_t = ((X_t - 1)/X_t) Y_t$.

2. The steady state

`(note: the notation here is the same that I use in the Matlab files hop_go.m and hop.m which solve for the steady state and the near-steady state dynamics of the model, see Section 7 below for details).`

One can always normalize the technology parameter A so that $Y = 1$ in steady state, so the trick is to express all the variables as a ratio to Y. In addition, the preference specification adopted implies that steady state hours worked are irrelevant for the dynamics around the steady state.

$$
\begin{aligned}
\pi &= 1 \\
R &= 1/\beta \\
\lambda &= (\beta - \gamma)/c \\
\lambda'' &= \left(\beta - \beta''\right)/c'' \\
F &= (1 - 1/X)Y \\
K &= \frac{\gamma\mu}{1-\gamma(1-\delta)}\frac{1}{X}Y \overset{\text{def}}{=} \zeta_1 Y \\
q &= \frac{\gamma\nu}{1-\gamma-(\beta-\gamma)m}\frac{1}{X}\frac{Y}{h} \overset{\text{def}}{=} \zeta_2 \frac{Y}{h} \\
q &= \frac{j'}{1-\beta}\frac{c'}{h'} \overset{\text{def}}{=} \zeta_3 \frac{c'}{h'} \\
q &= \frac{j''}{1-\beta''-m''\left(\beta-\beta''\right)}\frac{c''}{h''} \overset{\text{def}}{=} \zeta_4 \frac{c''}{h''} \\
b &= \beta m q h \\
c &= \frac{\mu+\nu}{X}Y - \delta K - (1-\beta) m q h \Longrightarrow c = \left(\frac{\mu+\nu}{X} - \delta\zeta_1 - (1-\beta) m \zeta_2\right) Y \overset{\text{def}}{=} \zeta_5 Y \\
b'' &= \beta m'' q h'' = \beta m'' \zeta_4 c'' \\
c'' &= w'' L'' - (1-\beta) m'' \zeta_4 c'' \\
w'' L'' &= (1-\alpha)(1-\mu-\nu) Y/X \overset{\text{def}}{=} s'' Y \\
w' L' + F &= (\alpha(1-\mu-\nu) + X - 1) Y/X \overset{\text{def}}{=} s' Y \\
c'' &= s'' Y - (1-\beta) m'' \zeta_4 c'' \Longrightarrow c'' = \frac{s''}{1+(1-\beta) m'' \zeta_4} Y \overset{\text{def}}{=} \zeta_6 Y \\
c' &= w' L' + F + (1-\beta)(m q h + m'' q h'') \Longrightarrow c' = (s' + (1-\beta)(m\zeta_2 + m''\zeta_4\zeta_6)) Y \overset{\text{def}}{=} \zeta_7 Y \\
h &= \frac{\zeta_2}{\zeta_3\zeta_7 + \zeta_4\zeta_6 + \zeta_2}, h' = \frac{\zeta_3\zeta_7}{\zeta_3\zeta_7 + \zeta_4\zeta_6 + \zeta_2}, h'' = \frac{\zeta_4\zeta_6}{\zeta_3\zeta_7 + \zeta_4\zeta_6 + \zeta_2}
\end{aligned}
$$

3. The log-linearised model

`(notice: the notation and the numbering of the equations are the same as in the Matlab file hop_go.m, see Section 7 below for details)`

It is convenient express the log-linearised model in terms of the following six blocks of equations. These are the same equations reported in Appendix A to the paper, but here I also keep L and λ in, before simulating the model. These are the equations contained in the file

1. aggregate demand

$$0 = \frac{c}{Y}\widehat{c}_t + \frac{c'}{Y}\widehat{c'}_t + \frac{c''}{Y}\widehat{c''}_t + \frac{I}{Y}\widehat{I}_t - \widehat{Y}_t \tag{E.1}$$

$$\widehat{c'}_t = \widehat{c'}_{t+1} - \widehat{R}_t + \widehat{\pi}_{t+1} \tag{E.2}$$

$$\beta\widehat{c}''_t = \beta''\widehat{c}''_{t+1} - (\beta - \beta'')\,\widehat{\lambda}''_t - \beta\widehat{R}_t + \beta''\widehat{\pi}_{t+1} \tag{E.3}$$

$$\beta\widehat{c}_t = \gamma\widehat{c}_{t+1} - (\beta - \gamma)\,\widehat{\lambda}_t - \beta\widehat{R}_t + \gamma\widehat{\pi}_{t+1} \tag{E.4}$$

$$\widehat{c}_t = \widehat{c}_{t+1} - \zeta\left(\widehat{Y}_{t+1} - \widehat{X}_{t+1} - \widehat{K}_t\right) + \psi\left(\widehat{I}_t - \widehat{K}_{t-1} - \gamma(\widehat{I}_{t+1} - \widehat{K}_t)\right) \tag{E.5}$$

2. housing market

$$\widehat{q}_t = \gamma_e\widehat{q}_{t+1} + (1-\gamma_e)\left(\widehat{Y}_{t+1} - \widehat{h}_t - \widehat{X}_{t+1}\right) + m_e\left(\widehat{\lambda}_t + \widehat{\pi}_{t+1} + \widehat{c}_{t+1}\right) + \widehat{c}_t - \widehat{c}_{t+1} \tag{E.6}$$

$$\widehat{q}_t = \gamma_h\widehat{q}_{t+1} + (1-\gamma_h)\left(\widehat{j}_t - \widehat{h}''_t\right) + m_h\left(\widehat{\lambda}''_t + \widehat{\pi}_{t+1}\right) + \widehat{c}''_t - \beta''\widehat{c}''_{t+1} \tag{E.7}$$

$$\widehat{q}_t = \beta\widehat{q}_{t+1} + (1-\beta)\left(\widehat{j}_t - \widehat{h}'_t\right) + \widehat{c'}_t - \beta\widehat{c'}_{t+1} \tag{E.8}$$

$$0 = h\widehat{h}_t + h'\widehat{h}'_t + h''\widehat{h''_t} \tag{E.9}$$

3. borrowing constraints

$$\widehat{b}_t = \widehat{q}_{t+1} + \widehat{h}_t + \widehat{\pi}_{t+1} - \widehat{R}_t \tag{E.10}$$

$$\widehat{b}''_t = \widehat{h}''_t + \widehat{q}_{t+1} + \widehat{\pi}_{t+1} - \widehat{R}_t \tag{E.11}$$

4. aggregate supply

$$\widehat{Y}_t = \widehat{A}_t + \nu\widehat{h}_{t-1} + \mu\widehat{K}_{t-1} + \alpha(1-\nu-\mu)\,\widehat{L'}_t + (1-\alpha)(1-\nu-\mu)\,\widehat{L''}_t \tag{E.12}$$

$$\widehat{Y}_t = \widehat{X}_t + \eta''\widehat{L}''_t - \left(\widehat{\lambda}''_t + \widehat{R}_t\right) \tag{E.13}$$

$$\widehat{Y}_t = \widehat{X}_t + \eta'\widehat{L}'_t + \widehat{c}'_t \tag{E.14}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} - \kappa\widehat{X}_t + \widehat{u}_t \tag{E.15}$$

5. Flows of funds / Evolution of state variables

$$\widehat{K}_t = \delta\widehat{I}_t + (1-\delta)\,\widehat{K}_{t-1} \tag{E.16}$$

$$\frac{b}{Y}\widehat{b}_t = \frac{c}{Y}\widehat{c}_t + \frac{qh}{Y}\left(\widehat{h}_t - \widehat{h}_{t-1}\right) + \frac{I}{Y}\widehat{I}_t + \frac{Rb}{Y}\left(\widehat{R}_{t-1} + \widehat{b}_{t-1} - \widehat{\pi}_t\right) - (1-s'-s'')\left(\widehat{Y}_t - \widehat{X}_t\right) \tag{E.17}$$

$$\frac{b''}{Y}\widehat{b}''_t = \frac{c''}{Y}\widehat{c}''_t + \frac{qh''}{Y}\left(\widehat{h}''_t - \widehat{h}''_{t-1}\right) + \frac{Rb''}{Y}\left(\widehat{R}_{t-1} + \widehat{b}''_{t-1} - \widehat{\pi}_t\right) - s''\left(\widehat{Y}_t - \widehat{X}_t\right) \tag{E.18}$$

6. Monetary policy rule and shock processes

$$\begin{aligned}
\widehat{R}_t &= (1-r_r)\left((1+r_\pi)\,\widehat{\pi}_t - r_x \widehat{X}_t\right) + r_r \widehat{R}_{t-1} + \widehat{\varepsilon}_{r,t} \\
\widehat{j}_t &= \rho_j \widehat{j}_{t-1} + \widehat{\varepsilon}_{j,t} \\
\widehat{u}_t &= \rho_u \widehat{u}_{t-1} + \widehat{\varepsilon}_{u,t} \\
\widehat{A}_t &= \rho_A \widehat{A}_{t-1} + \widehat{\varepsilon}_{A,t}
\end{aligned}$$

where I have defined the following constants: $\gamma_e \equiv \gamma + m(\beta-\gamma)$, $\gamma_h \equiv \beta'' + m''\left(\beta-\beta''\right)$, $m_e \equiv m(\beta-\gamma)$, $m_h \equiv m''\left(\beta-\beta''\right)$, $\zeta \equiv 1-\gamma(1-\delta)$, $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta$.

3.1. Some derivations

3.1.1. Derivation of $(E5)$

Start from the two equations in the text

$$\begin{aligned}
u_t &= \frac{1}{c_t}\left(1+\psi\left(\frac{I_t}{K_t}-\delta\right)\right) \\
u_t &= \frac{1}{c_t}\left(\psi\left(\frac{I_t}{K_t}-\delta\right)\frac{I_t}{K_t} - \frac{\psi}{2}\left(\frac{I_t}{K_t}-\delta\right)^2\right) + \gamma E_t\left(\frac{\mu Y_{t+1}}{c_{t+1}X_{t+1}K_t} + u_{t+1}(1-\delta)\right)
\end{aligned}$$

observe that appropriately combined the two equations yields:

$$1+\frac{\psi}{\delta}\left(\frac{I_t}{K_{t-1}}-\delta\right) = E_t\left[\frac{c_t}{c_{t+1}}\gamma\left(\frac{\mu Y_{t+1}}{X_{t+1}K_t} + 1 - \delta + \frac{\psi}{\delta}\underbrace{\left(\frac{I_{t+1}}{K_t}-\delta\right)}_{\text{zero in ss}}\underbrace{\left(\frac{1}{2}\left(\frac{I_{t+1}}{K_t}+\delta\right)+1-\delta\right)}_{\text{1 in steady state}}\right)\right] \qquad \text{(xx)}$$

Notice that in steady state both sides of the equations are 1.

$$\text{steady state: } \delta = I/K$$

$$\text{steady state:} \frac{\mu Y}{XK} + 1 - \delta = \frac{1}{\gamma} --> \frac{\gamma\mu Y}{XK} = 1-\gamma(1-\delta)$$

Inspection of the combination of the two equations shows that the (LHS) and (RHS) of equation (xx) are, appropriately linearized:

$$LL\_LHS = \frac{\psi}{\delta}\frac{I}{K}(I_t - K_t) = \psi(I_t - K_t)$$

$$LL\_RHS = c_t - c_{t+1} + (1-\gamma(1-\delta))(Y_{t+1} - X_{t+1} - K_t) + \gamma\psi(I_{t+1} - K_t)$$

Combine LL_LHS and LL_RHS to obtain:

$$\widehat{c}_t = \widehat{c}_{t+1} - \zeta\left(\widehat{Y}_{t+1} - \widehat{X}_{t+1} - \widehat{K}_t\right) + \psi\left(\widehat{I}_t - \widehat{K}_{t-1} - \gamma(\widehat{I}_{t+1} - \widehat{K}_t)\right)$$

3.1.2. Housing adjustment costs

Assume there is a quadratic cost associated with increasing or decreasing the amount of housing h for each agent. Denoting with AC_t the adjustment cost, the budget constraints of the three agents become now:

$$\begin{aligned}
Y_t + b_t &= c_t + q_t(h_t - h_{t-1}) + k_t - (1-\delta)k_{t-1} + R_{t-1}b_{t-1}/\pi_t + w_t' l_t' + w_t'' l_t'' + AC_t \\
w_t' l_t' + b_t' &= c_t' + q_t\left(h_t' - h_{t-1}'\right) + R_{t-1}b_{t-1}'/\pi_t + AC_t' \\
w_t'' l_t'' + b_t'' &= c_t'' + q_t\left(h_t'' - h_{t-1}''\right) + R_{t-1}b_{t-1}''/\pi_t + AC_t''
\end{aligned}$$

Assume that in order to adjust the stock from h_{t-1} to h_t, agents must pay a cost of the form:

$$AC_t = \frac{\phi q_t}{2}\left(\frac{h_t - h_{t-1}}{h_{t-1}}\right)^2 h_{t-1}$$

Market clearing implies now:

$$Y_t = c_t + c'_t + c''_t + k_t - (1-\delta)\,k_{t-1} + \sum AC$$

however $\sum AC$ (sum of quadratic terms) disappears from the linear approximation (equation 1_L). What changes are the three first order conditions for the agents for the optimal choice of h. When agents increase or decrease h_t, they also pay the adjustment cost. This implies:

$$\begin{aligned}
\frac{1}{c_t}\left(q_t + \phi q_t\left(\frac{h_t - h_{t-1}}{h_{t-1}}\right)\right) &= \frac{\gamma}{c_{t+1}}\left(q_{t+1} + \frac{\phi}{2}q_{t+1}\left(\frac{h_{t+1}^2 - h_t^2}{h_t^2}\right) + \nu\frac{Y_{t+1}}{h_t}\right) + \lambda_t m_t q_{t+1} \\
\frac{1}{c'_t}\left(q_t + \phi' q_t\left(\frac{h'_t - h'_{t-1}}{h'_{t-1}}\right)\right) &= \frac{j}{h'_t} + \frac{\beta}{c'_{t+1}}\left(q_{t+1} + \frac{\phi'}{2}q_{t+1}\left(\frac{h'^2_{t+1} - h'^2_t}{h'^2_t}\right)\right) \\
\frac{1}{c''_t}\left(q_t + \phi'' q_t\left(\frac{h''_t - h''_{t-1}}{h''_{t-1}}\right)\right) &= \frac{j''}{h''_t} + \frac{\beta''}{c''_{t+1}}\left(q_{t+1} + \frac{\phi''}{2}q_{t+1}\left(\frac{h''^2_{t+1} - h''^2_t}{h''^2_t}\right)\right) + \lambda''_t m''_t q_{t+1}
\end{aligned}$$

For instance when the Entrepreneur buys h_t the cost is given by q_t plus the cost of varying the size from h_{t-1} to h_t. The benefit is given by extra production in the future, possible future adjustment cost avoided, relaxation of the borrowing constraint.

In the log-linear approximation of the model, the terms below need to be added to E6, E7 and E8 (see file `hop_go.m`):

$$... + \phi\left(\widehat{h}_t - \widehat{h}_{t-1}\right) ... = ... + \gamma\phi\left(\widehat{h}_{t+1} - \widehat{h}_t\right) \qquad \text{(E6\_ac)}$$

$$... + \phi''\left(\widehat{h''}_t - \widehat{h''}_{t-1}\right) = ... + \beta''\phi''\left(\widehat{h''}_{t+1} - \widehat{h''}_t\right) \qquad \text{(E7\_ac)}$$

$$... + \frac{\phi'}{h'}\left(h\widehat{h}_{t-1} + h''\widehat{h''}_{t-1} - h\widehat{h}_t - h''\widehat{h''}_t\right) = ... + \frac{\beta\phi'}{h'}\left(h\widehat{h}_t + h''\widehat{h''}_t - h\widehat{h}_{t+1} - h''\widehat{h''}_{t+1}\right) \qquad \text{(E8\_ac)}$$

4. Some notes on model estimation

4.1. The restricted VAR representation of the model and of the variables that one wants to take to the data

Denote with $\mathbf{x}$ the vector collecting the variables that you take to the data (like output and house prices), with $\mathbf{y}$ those that appear in the model but you do not want in estimation (like hours and capital), with $\mathbf{u}$ the vector of shocks in the model. In the model, the shocks are uncorrelated, so $E\mathbf{uu}'$ is a diagonal matrix.

The decision rules for the model can then be written as:[1]

$$
\begin{aligned}
\dim \mathbf{x} &= 4\times 1 \\
\dim \mathbf{y} &= 20\times 1 \\
\dim \mathbf{u} &= 4\times 1 \\
\mathbf{x}_t &= \mathbf{P}_{4\times4}\mathbf{x}_{t-1} + \mathbf{Q}_{4\times20}\mathbf{y}_{t-1} + \mathbf{R}_{4\times4}\mathbf{u}_t \\
\mathbf{y}_t &= \mathbf{S}_{20\times4}\mathbf{x}_{t-1} + \mathbf{U}_{20\times20}\mathbf{y}_{t-1} + \mathbf{V}_{20\times4}\mathbf{u}_t
\end{aligned}
$$

Given that we do not observe $\mathbf{y}$, we want to solve this system of equation for $\mathbf{x}_t$ only. Denote L the lag operator, and $\mathbf{I}$ a conformable identity matrix. Then:

$$
\begin{aligned}
(\mathbf{I}-\mathbf{P}L)\,\mathbf{x}_t &= \mathbf{Q}L\mathbf{y}_t + \mathbf{R}\mathbf{u}_t \\
(\mathbf{I}-\mathbf{U}L)\,\mathbf{y}_t &= \mathbf{S}L\mathbf{x}_t + \mathbf{V}\mathbf{u}_t
\end{aligned}
$$

next

$$
\begin{aligned}
\mathbf{y}_t &= (\mathbf{I}-\mathbf{U}L)^{-1}\mathbf{S}L\mathbf{x}_t + (\mathbf{I}-\mathbf{U}L)^{-1}\mathbf{V}\mathbf{u}_t \\
(\mathbf{I}-\mathbf{P}L)\,\mathbf{x}_t &= \mathbf{Q}L\left((\mathbf{I}-\mathbf{U}L)^{-1}\mathbf{S}L\mathbf{x}_t + (\mathbf{I}-\mathbf{U}L)^{-1}\mathbf{V}\mathbf{u}_t\right) + \mathbf{R}\mathbf{u}_t \\
&= \mathbf{Q}L\,(\mathbf{I}-\mathbf{U}L)^{-1}\mathbf{S}L\mathbf{x}_t + \mathbf{Q}L\,(\mathbf{I}-\mathbf{U}L)^{-1}\mathbf{V}\mathbf{u}_t + \mathbf{R}\mathbf{u}_t \\
\left(\mathbf{I}-\mathbf{P}L-\mathbf{Q}L\,(\mathbf{I}-\mathbf{U}L)^{-1}\mathbf{S}L\right)\mathbf{x}_t &= \left(\mathbf{Q}L\,(\mathbf{I}-\mathbf{U}L)^{-1}\mathbf{V}+\mathbf{R}\right)\mathbf{u}_t \qquad (1) \\
A(L)\,\mathbf{x}_t &= (\mathbf{R}+B(L))\,\mathbf{u}_t \qquad (2)
\end{aligned}
$$

It appears clear that if $E\mathbf{uu}' = \Sigma_{\mathbf{u}}$, a diagonal matrix, the impact effect on $\mathbf{x}_t$ of a shock to the vector $\mathbf{u}$ will be given by $\mathbf{R}$: on the LHS of (1), the only contemporaneous term on $\mathbf{x}$ is given by the $\mathbf{I}$ matrix, whereas on the RHS the only contemporaneous term on $\mathbf{u}$ is $\mathbf{R}$. That is

$$\frac{d\mathbf{x}_t}{d\mathbf{u}_t} = \mathbf{R}$$

In general, $\mathbf{R}$ will not be triangular. To have a meaningful comparison of the model impulse responses with the data impulse responses, we want to obtain a linear combination of the model shocks that shares the same triangular impact effect of the empirical VAR.

To this effect, rewrite equation 2 as

$$
\begin{aligned}
A(L)\,\mathbf{x}_t &= \left(\mathbf{R}^{-1}\mathbf{R}+\mathbf{R}^{-1}\mathbf{R}B(L)\,\mathbf{R}^{-1}\right)\mathbf{R}\mathbf{u}_t \\
A(L)\,\mathbf{x}_t &= \left(\mathbf{I}+B(L)\,\mathbf{R}^{-1}\right)\mathbf{R}\mathbf{u}_t \equiv \left(\mathbf{I}+B(L)\,\mathbf{R}^{-1}\right)\mathbf{Z}\boldsymbol{\varepsilon}_t
\end{aligned}
$$

[1] If $\mathbf{x}$ is not an endogenous state variable, the corresponding row of $\mathbf{P}$ or $\mathbf{S}$ will have all zeros. In our case, the vector $\mathbf{x}$ includes (R,π,q,Y). Of those, only the interest rate R is an endogenous state variable (the lagged interest rate appears in fact in the Taylor rule).

so that

$$\begin{aligned}\mathbf{Z}\varepsilon &= \mathbf{Ru} \\ \mathbf{Z}\boldsymbol{\Sigma}_{\varepsilon}\mathbf{Z}' &= \mathbf{R}\boldsymbol{\Sigma}_{\mathbf{u}}\mathbf{R}'\end{aligned}$$

where $\boldsymbol{\Sigma}_{\varepsilon}$ is diagonal. We want $\mathbf{Z}$ to be a triangular matrix, so that

$$\frac{d\mathbf{x}_t}{d\varepsilon_t} = \mathbf{Z}$$

gives us model impulse responses as those in the data-based VAR. To this purpose, the Choleski factorization of $\mathbf{R}\boldsymbol{\Sigma}_{\mathbf{u}}\mathbf{R}'$ produces the required triangular matrix $\mathbf{Z}$.

Further details are in the Matlab file `irf_4by4_compare_model_to_var.m`

4.2. Test of equality between the model and the data

`(notice: the notation here follows lsq1.m, see Section 7 for details)`

Denote with $\boldsymbol{\Psi}(\boldsymbol{\zeta})$ the model κ impulse responses, obtained from the reduced form of the model by ordering and orthogonalizing the impulse responses as in the VAR.

Let $\widehat{\boldsymbol{\Psi}}$ denote the $n \times 1$ vector of empirical estimates of the VAR impulse responses.[2] I include the first h elements of each impulse response function, excluding those that are zero by the recursiveness assumption. The estimate of ζ, a $k \times 1$ vector of parameters to estimate, solves:

$$\min_{\boldsymbol{\zeta}} \left(\boldsymbol{\Psi}(\boldsymbol{\zeta}) - \widehat{\boldsymbol{\Psi}}\right)' \boldsymbol{\Phi} \left(\boldsymbol{\Psi}(\boldsymbol{\zeta}) - \widehat{\boldsymbol{\Psi}}\right) = \mathsf{m}(\boldsymbol{\zeta})' \boldsymbol{\Phi} \mathsf{m}(\boldsymbol{\zeta})$$

where $\boldsymbol{\Phi}_{n\times n}$ is a (diagonal) weighting matrix that we will be discussed below. Typically, the weighting matrix would be $\boldsymbol{\Phi} = \boldsymbol{\Upsilon}^{-1}$, the inverse of matrix with the sample variances of the impulse responses on the main diagonal.

Our estimation criterion prefers a matrix $\boldsymbol{\Phi} = \boldsymbol{\Omega}\boldsymbol{\Upsilon}^{-1}$, where $\boldsymbol{\Omega}_{n\times n}$ is a diagonal matrix of weights that give more importance to some impulse responses than others. Under these circumstances, the covariance matrix for the estimator $\widehat{\boldsymbol{\zeta}}_{k\times 1}$ that minimizes the criterion function above is (see Greene, third edition, page 526):

$$V_A = \left(G'\boldsymbol{\Phi}G\right)^{-1} G'\boldsymbol{\Phi}\boldsymbol{\Upsilon}\boldsymbol{\Phi}G \left(G'\boldsymbol{\Phi}G\right)^{-1}$$

where $G_{n\times k}$ is the gradient of $\mathsf{m}(\boldsymbol{\zeta})$ and has dimension number of moment conditions $\times$ number of parameters to estimate).

It turns out that the above expression simplifies greatly in Matlab, where the routine `lsqnonlin.m` requires the user-defined function to compute the vector-valued function, rather than compute the value $\mathsf{m}'\boldsymbol{\Phi}\mathsf{m}$ (the "sum of squares"):

$$\mathsf{j}(\boldsymbol{\zeta}) = \mathsf{m}(\boldsymbol{\zeta})' \boldsymbol{\Phi}^{1/2}$$

so that $\dim(\mathsf{j}) = n \times 1$. In this case, it turns out that, defining the derivative matrix of $\mathsf{j}(\boldsymbol{\zeta})$ with $D_{n\times k} = \boldsymbol{\Phi}^{1/2}\frac{\partial \mathsf{m}(\boldsymbol{\zeta})}{\partial \boldsymbol{\zeta}} = \boldsymbol{\Phi}^{1/2}G$, the covariance matrix for the estimator $\widehat{\boldsymbol{\zeta}}$ can be written as

$$\begin{aligned} V_A &= \left(G'\boldsymbol{\Phi}G\right)^{-1}\left(G'\boldsymbol{\Phi}\boldsymbol{\Upsilon}\boldsymbol{\Phi}G\right)\left(G'\boldsymbol{\Phi}G\right)^{-1} = \\ &= \left[\left(D'\boldsymbol{\Phi}^{-\frac{1}{2}}\right)\boldsymbol{\Phi}\left(\boldsymbol{\Phi}^{-\frac{1}{2}}D\right)\right]^{-1}\left[\left(D'\boldsymbol{\Phi}^{-\frac{1}{2}}\right)\boldsymbol{\Phi}\boldsymbol{\Upsilon}\boldsymbol{\Phi}\left(\boldsymbol{\Phi}^{-\frac{1}{2}}D\right)\right]\left[\left(D'\boldsymbol{\Phi}^{-\frac{1}{2}}\right)\boldsymbol{\Phi}\left(\boldsymbol{\Phi}^{-\frac{1}{2}}D\right)\right]^{-1}\end{aligned}$$

using the fact that $\boldsymbol{\Phi}$ and $\boldsymbol{\Upsilon}$ are diagonal matrices, and that $\boldsymbol{\Upsilon} = \boldsymbol{\Omega}\boldsymbol{\Phi}^{-1}$, the above simplifies to

$$V_A = \left[D'D\right]^{-1}\left[D'\boldsymbol{\Omega}D\right]\left[D'D\right]^{-1}$$

[2] $n = m^2 \times h - z$, where m is the number of variables included in the VAR, h is the horizon until which the impulse responses one wants to match, and z are the elements of the vector which are zero by assumption (for instance, because the VAR imposes zero restrictions thanks so some Choleski ordering)

5. Some notes on the model in Appendix C

To keep the problem as general as possible, we study the problem of an agent who uses the durable asset both as good, as collateral and as a factor of production.

Write the momentary utility function as:

$$U = u(c, h) = \frac{\left(c_t h_t^j\right)^{1-\rho}}{1-\rho} \text{ when } \rho > 0$$

in the paper, log utility arises when $\rho = 1$. With this formulation:

$$\begin{aligned} u_h &= \left(ch^j\right)^{-\rho} jch^{j-1} \\ u_c &= \left(ch^j\right)^{-\theta} h^j \\ \frac{u_h}{u_c} &= j\frac{c}{h} \end{aligned}$$

Normalizing $q = 1$, the budget constraint is

$$c_t = A_t h_{t-1}^{\nu} + b_t - Rb_{t-1} - (h_t - (1-\delta) h_{t-1})$$

where b_{t-1} denotes the outstanding bond obligations for each agent. The stochastic process for A_t is Markov.

b is bounded by some fraction of available collateral. That is:

$$b_t < mh_t$$

The Lagrangean is therefore:

$$\begin{aligned} U = \; & u\left(\left(A_t h_{t-1}^{\nu} + b_t - Rb_{t-1} - (h_t - (1-\delta) h_{t-1})\right), h_t\right) + \\ & \gamma u\left(\left(A_{t+1} h_t^{\nu} + b_{t+1} - Rb_t - (h_{t+1} - (1-\delta) h_t)\right), h_{t+1}\right) + ... \\ & -\lambda_t (b_t - mh_t) - ... \end{aligned}$$

We can write the set of Euler equations choosing b and h

$$\begin{aligned} u_{c,t} &= \beta R u_{c,t+1} + \lambda_t \\ u_{ct} &= u_{ht} + \beta\left((1-\delta) + \nu E_t A_{t+1} h_t^{\nu-1}\right) u_{c,t+1} + \lambda_t m \end{aligned}$$

or

$$(1-m) u_{ct} = u_{ht} + \beta\nu A_t h_t^{\nu-1} u_{ct+1} + \beta(1-\delta-mR) u_{c,t+1}$$

know that we will have in absence of shocks, for $\gamma R < 1$

$$\begin{aligned} 1 &= \frac{u_h}{u_c} + \beta\left((1-\delta) + \nu A_t h^{\nu-1}\right) + (1-\gamma R) m \\ c &= Ah^{\nu} - (R-1) b \end{aligned}$$

Hence the steady state value of h is the solution to the equation above, that is

$$1 - \beta(1-\delta) - (1-\beta R) m = \frac{\gamma\nu A}{h^{1-\nu}} + j\left(\frac{A}{h^{1-\nu}} - (R-1) m\right)$$

6. The notation used in the paper

1. **UPPERCASE LATIN**

   A technology

   B, B' nominal debt and borrowing in basic model ($\overline{B}$ in appendix B)

   E expectation operator

   F_t are lump-sum profits received from the retailers

   H total supply of housing

   I real investment

   K capital

   L, L', L'' are labor supply

   M nominal money

   P price index

   P^W wholesale price index

   Q nominal asset price (and quarter indicator in dates)

   R nominal interest rate (and Funds rate in VAR)

   S marginal product of housing

   T', T'' transfers

   W' nominal wage

   $X_t \equiv P_t/P_t^w$ denotes the markup of final over intermediate goods

   Y intermediate (final) output (and GDP in VAR)

   Y^f final good.

2. **lowercase latin**

   b, b', b'' real borrowing, lending

   c, c', c'' real consumption

   d differentiation sign

   e_R, e_j, e_u, e_A : the four shocks

   h, h', h'' housing holdings

   j housing weight

   k time subscript in $t + k$

   m, m'' loan-to-value ratios

   n to denote vector length in estimation of the model

   q house prices (and in VAR)

   r_R, r_Y, r_π, r_q Taylor rule parameters

   rr real rate

   s', s'' income shares of patient and impatient households

   t time

u inflation shock

v shadow price of capital

w', w'' real wage

z index for retailers

3. **lowercase greek**

α patient household wage share

β, β'' discount factor for households

γ discount factor for entrepreneur

$\gamma_e = m\beta + (1-m)\gamma$

$\gamma_h = \beta'' + m'' \left(\beta - \beta''\right)$

δ depreciation rate for K

ε elasticity of demand for final goods

$\boldsymbol{\zeta}$ vector of parameters to estimate

η labor disutility

θ price rigidity

$\iota = (1-\beta)\, h/h'$ constant in linearized housing demand

$\iota'' = (1-\beta)\, h''/h'$ constant in linearized housing demand

κ Phillips curve slope

λ Lagrange multiplier on borrowing constraint

μ capital share in production

ν housing share in production

$\xi_{K,t} = \psi \left(I_t/K_{t-1} - \delta\right)^2 K_{t-1}/2\delta$ capital installation cost

$\xi_{he,t} = \phi_e q_t \left(\Delta h_t/h_{t-1}\right)^2/2$ housing adjustment cost

π inflation

ρ_A, ρ_j, ρ_u shocks autocorrelation

ρ RRA in the appendix

σ_e,$\sigma_A, \sigma_j, \sigma_u$ and variance

ϕ, ϕ_e, ϕ_h housing adjustment cost

χ weight on money demand

ψ capital adjustment cost

$\omega = \left(\beta'' - m''\beta''\right)/\left(1 - m''\beta\right)$ a constant in log-linearizations

4. **UPPERCASE GREEK**

Δ the first difference operator

$\boldsymbol{\Upsilon}$, the matrix with the sample variances of the VAR impulse responses on the main diagonal

$\boldsymbol{\Phi}$ weighting matrix

$\boldsymbol{\Psi}(\boldsymbol{\zeta})$ the model κ impulse responses

$\widehat{\mathbf{\Psi}}$ be the $n \times 1$ vector of empirical estimates of the VAR impulse responses

$\mathbf{\Omega}$ is a $n \times n$ diagonal matrix of weights that gives a weight four times large

7. Further details on the replication files

This section gives a short description of the files necessary to solve, simulate and estimate the model from the paper "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle"

7.1. Installation and Software Requirements

a) In order to estimate the data VAR you need Rats (`www.estima.com`)

b) In order to solve and simulate the model and estimate its structural parameters and the policy frontiers you need Matlab and the Harald Uhlig's toolkit available at http://www.wiwi.hu-berlin.de/wpol/html/toolkit/new/toolkit2.zip (for convenience, the files have been already put inside the _add_ons directory in the directory `uhlig_original`). You will also need to add two sliglty different version of the files `solve.m` and `do_it.m`, that I have slightly customized creating `solve3.m` and `do_it3.m` (see respective files for details: in brief, both files do not print out some of the output on the screen, unlike the original files), which are in the subdirectory `_add_ons`.

c) Before running the simulations you should unzip all the files into some directory in your computer and you should add all the directories into Matlab search path. I have tested them on a directory like c:\e\houseprices_AER.

d) The simulation files are known to work with Matlab 7. With minor exceptions, they should work also on Matlab 6.5. Let me know if you encounter problems on earlier versions of Matlab.

Below is a description of the folders in the replication directory.

7.2. VAR_AER

This directory contains the file needed to replicate the VAR of Section I as well as the impulse responses of the consumption VAR of Section III.D. The masterfiles in this directory are the RATS (www.estima.com) program files `BENCHMARK_VAR.PRG` and `CONSUMPTION_VAR.PRG.` The exact data definitions and sources can be found in the two excel files also contained in this directory. Additional explanations are contained inside the files themselves. Before running the programs, you should make sure the files are in the correct directory.

- `BENCHMARK_VAR.PRG:`

  Rats program needed to:

  1) Generate and store the impulse responses and standard errors of the VAR of figure 1 in the paper

  2) Estimate the coefficients of the taylor rule of the "econometric methodology section of the paper

  The impulse responses of the 4 variables to the 4 shocks and their upper and lower band are stored in the ascii files `RE.TX REU.TX REL.TX`

- `CONSUMPTION_VAR.PRG`

  Rats program needed to generate and store the impulse responses and standard errors of the consumption VAR of figure 3 in the paper

  The impulse responses of house prices and consumption to a house price shock and their upper and lower band are stored in the ascii files `Q_SHOCK.TX Q_SHOCK_LO.TX QSHOCK_UP.TX`

- The two excel files `US_MONTHLY_DATA.XLS US_QUARTERLY_DATA.XLS` contain the original series plus other series which were used in the specification search. These series are imported in VAR at the beginning of the file benchmark.prg. The final series which are used in the estimation are (Y, P, Q, R, CE). They are stored with this name (alongside all other series) in the RATS data file `USDATA_ALL.RAT (US_DATA.RAT` is a transit data file).

- In order to detrend the data with the bandpass filter, I use the code `BPFILTER.SRC` available in the Rats public domain. See subfolder `AUXILIARY_VAR_FILES`

7.3. SIMULATION_AER

This directory contains the files needed to replicate the model impulse responses plus other model features. The masterfile is `hop.m`. All other files are called by `hop.m`, and do not work in isolation.

- `HOP.M`

  This file defines options and calibrated parameters for the paper. You can modify one of the examples in the file to run sensitivity analysis on the model.

  For instance, uncomment the line below example (A) (the line that says hop_basic.m) to replicate the impulse responses of Figure 1. Or uncomment all the lines below example (1) to run two models (one with and one without price rigidity) and to compare them. Each of the examples should be run in isolation from the others.

- `HOP_GO.M`

  This file contains all the equations of the linearized model in the matrices AA, BB and so on... (the notation is that of Uhlig, 1999).

- `HOP_BASIC.M`: shows cumulative output drop to monetary shock in basic model.
- `HOP_HOUSING_SHOCK.M`: compares model vs data response of consumption and house prices to a housing preference shock.
- `HOP_INFLATION_SHOCK.M` : compares IRF from the model inflation shock with the data inflation shock.
- `IRF_4BY4.M` : shows irf's graphs in 4X4 diagram for more than one shock.
- `IRF_4BY4_CHOLESKI_THE_MODEL.M` : re-orders model impulse responses in a Choleski fashion.
- `IRF_4BY4_COMPARE_MODEL_TO_VAR.M`: compares impulse responses from the data-VAR with those from the model.
- `IRF_HOUSING_SHOCK.M` : modified irf for house price shock on consumption.
- `IRF_INFLATION_SHOCK.M` : modified responses for an inflation shock on output.
- `IRF_MONETARY_SHOCK.M` : shows in same graph irfs for same monetary shock from more than one calibration.
- `LABEL_AXIS.M` : adds labels to plots of impulse response functions.
- `SCALE_AXIS.M` : rescales axis so that each variable has same min and max across all IRF.
- `VOLATILITY_GIMME.M` : These lines below calculate volatility of series for each set of calibrated parameters

7.4. ESTIMATION_AER

This directory contains the files that are needed to estimate the structural parameters model by minimizing the distance between the data impulse responses and the impulse responses of the model, κ as in the data. The masterfile is `lsq1.m`. All other files do not work in isolation. See help in lsq1.m to see in which order the files are called.

- `LSQ1.M`

  This file defines in the vector VARIB the variables that one wants to estimate, sets the lower and upper bound and then calls the function file `LOSS_LSQ` that evaluates the distance between the model and the data for each combination of the parameters to estimate. Once the parameters are estimated, LSQ1.M estimates the standard errors of the estimated parameters. This file uses the Matlab optimization function `lsqnonlin` to minimize the distance between the model and the data. The option CHOLESKI_THE_MODEL=(0,1) chooses whether or not to orthogonalize the model impulse responses.

- `LOSS_LSQ.M`

  This file evaluates the loss function. It does so by calculating first the vector valued distance (GAP0) between the VAR impulse responses (called RE) and the model impulse responses (called MO_0 if the model shocks are orthogonalized, and RE_0 if the model shocks are not orthogonalized).

- `CHOLESKI_THE_MODEL2.M`

  Orthogonalizes the model impulse responses and saves them in the object MO_0 with the option

  CHOLESKI_THE_MODEL=1; otherwise, model impulse responses are saved in the object RE_0 and are not orthogonalized.

- `CALIBSET_EST.M` : assigns values to all the parameters that are calibrated rather then estimated.

7.5. FRONTIER_AER

This directory contains the files that are needed to calculate the policy frontiers (Taylor curves) of the model for different values of the parameters of the Taylor rule. The masterfile is `fro1.m.` All other files do not work in isolation, except `plot_fro.m` which generates the plots of figure 6 to 8 in the paper with the data saved in the files data1.mat to data10.mat.

Notice that fro1.m takes about 1 hour on my Pentium 3 with 1 giga of RAM to calculate all the frontiers.

- `FRO1.M` : calculates policy frontiers for the paper. This file uses the Matlab optmization function `fmincon` to calculate the parameter of the Taylor rule that minimize the policymaker loss function (described in LOSS_FRO) for each relative weight on output and inflation.
- `CALIBSET_FRO.M` : assigns values for the calibrated parameters in policy frontier analysis
- `LOSS_FRO.M` : function file that calculates loss function for each policy
- `NONLCON_SDR.M` : imposes a nonlinear constraint on the standard deviation of the interest rate in policy frontier
- `PLOT_FRO.M` : plots the several policy frontiers obtained with fro1.m

7.6. APPENDIXC_AER

This directory contains the files that are needed to replicate the results contained in the appendix C of the paper available on the AER website or in the BC working paper version of the paper. The masterfile is `kfi1.m`. All other files do not work in isolation, except `kfi_plotter.m` which generates the plots of the figures 1 to 4 in the appendix with the data saved in the appropriate mat files.

Notice that kfi1.m takes about 10 hours to solve all the models in the appendix. The cases of Figure 1 and 2 take only a few minutes though, if you want to replicate the examples of Figure 1 and 2 comment the lines below the code of Figure 2.

- `KFI1.M` : set-up of model of consumption and housing choice under uncertainty
- `KFI_GO.M` : calculates policy functions for the baseline problem described in kfi1.m

- `KFI_HOWARD.M` : Howard improvement algorithm on the file KFI_GO.M
- `KFI_PLOTTER.M` : generates plots similar to those contained in the appendix C
- `KFI_SIM.M` : Simulates the economy (generating artificial time series) once the policy functions of a given model are calculated by kfi_go.m
- `KFI_SS_HOUSE`: function file that calculates non-stochastic steady state holdings of h in the model of appendix C

7.7. _ADD_ONS

This directory includes some auxiliary Matlab files. The files `do_it3` and `solve3.m` are slightly modified versions of Harald Uhlig's toolkit original files. The directory `uhlig_original` contains the original Uhlig's toolkit files.

The Hansen and Sargent's file `doublej.m` solves a discrete Lyapunov equation and is needed to calculate the theoretical variances generates by the model. It is also included in the `_ADD_ONS` directory.

The function files `rows.m cols.m` are Matlab function files calculate rows and columns of a given matrix.

The function file `suptitle.m` (taken from Matlab public domain) adds a large title to a Matlab figure.

The function files `vec.m` and `tschk.m` are from Jeff Fuhrer from Boston Fed and are described in the help file.

Other remarks In the calculations of the policy frontiers or in the estimation of the structural parameters of the model, sometimes the maximization algorithms might encouter discountinuities that result in Matlab's warning messages such as `divideByZero` or similar. For this reason, in my startup file in Matlab I have following options, which avoid splashing on the screen the warnings below.

warning off MATLAB:divideByZero
warning off MATLAB:singularMatrix
warning off MATLAB:polyfit:RepeatedPointsOrRescale
warning off MATLAB:dispatcher:InexactMatch