



## Financial business cycles <sup>☆</sup>



Matteo Iacoviello

Federal Reserve Board, 20th and C St. NW, Washington, DC 20551, United States

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### ABSTRACT

Using Bayesian methods, I estimate a DSGE model where a recession is initiated by losses suffered by banks and exacerbated by their inability to extend credit to the real sector. The event triggering the recession has the workings of a redistribution shock: a small sector of the economy – borrowers who use their home as collateral – defaults on their loans. When banks hold little equity in excess of regulatory requirements, the losses require them to react immediately, either by recapitalizing or by deleveraging. By deleveraging, banks transform the initial shock into a credit crunch, and, to the extent that some firms depend on bank credit, amplify and propagate the shock to the real economy. I find that redistribution and other financial shocks that affect leveraged sectors accounts for two-thirds of output collapse during the Great Recession.

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## 1. Introduction

In this paper I estimate, using Bayesian methods, a model with banks and financially constrained households and firms. I present a basic model which conveys the main ideas. I then take a richer version of this model to the data, estimate it using Bayesian methods, and use it to provide an accounting of the role played by different financial shocks and frictions during the financial crisis.

The main questions that I ask are: (1) How much can redistributions of wealth – such as those that take place when borrowers default on their debts – disrupt the credit intermediation process? (2) Can changes in credit standards affect business cycles? (3) How important are shocks to asset prices for business fluctuations? To answer these questions, I add financial frictions on banks, on households, and on firms to an otherwise standard RBC model and conduct a horse race between familiar shocks (a shock to the consumption/leisure margin, shocks to technology) and not-so-familiar ones. The not-so-familiar ones are redistribution shocks<sup>1</sup> (transfers of wealth from savers to borrowers that take place in the event of default); credit squeezes (changes in maximum loan-to-value ratios); and asset price shocks (changes in the value of

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E-mail address: [matteo.iacoviello@frb.gov](mailto:matteo.iacoviello@frb.gov).

<sup>1</sup> Throughout the paper, I use the terms “redistribution shocks”, “repayment shocks” and “default shocks” interchangeably.

collateral). These “financial shocks” were arguably at the core of the last recession. More generally, financial factors were at the core of at least two of the last three recessions in the United States (the 1990–1991 recession and the Great Recession of 2007–2009). Yet a large class of estimated dynamic equilibrium models either ignore financial frictions, or consider one set of financial frictions independently from others. While this approach might be useful for building intuition, it eludes a proper quantification of the role of financial factors in business fluctuations, especially when several sets of financial frictions reinforce and amplify each other.

The estimation of the model parameters and structural shocks gives large prominence to financial business cycles. I find that financial shocks account for two-thirds of the decline in private GDP during the 2007–2009 recession, and they also play an important, although less sizeable, role during other recessions. Although model parameters and shocks are jointly estimated, my approach has also the natural interpretation of a business cycle accounting exercise. This happens because some of the key shocks are directly used as observables at the estimation stage, so that their filtering is decoupled from the estimation of the rest of the model’s structural parameters.<sup>2</sup>

At the core of the paper is the idea that business cycles are financial rather than real. That is, rather than originated and propagated by changes in technology, business cycles are mostly caused by disruptions in the flow of resources between different groups of agents. In the model economy of this paper, these disruptions take place when a group of agents defaults on its obligations, therefore paying back less than contractually agreed. Or when credit limits are relaxed or tightened either in response to changes in asset prices or for some other exogenous reason. Of course, many of the stories told here resemble familiar accounts of the Great Recession: the bursting of the housing bubble merely changed the value of houses in units of consumption, yet it led to a wave of defaults and to a severe crisis in the financial sector. The ensuing problems of the financial institutions that owned mortgages lead to a reduction in the supply of credit to all sectors of the economy. Many of these ideas are all familiar. The novel elements are the financial shocks, and the estimation.<sup>3</sup>

Several of the ideas and modeling devices in this paper build on an important tradition in macroeconomic modeling that treats banks as intermediaries between savers and borrowers. Recent contributions include Brunnermeier and Sannikov (2014), Angeloni and Faia (2013), Gerali et al. (2010), Kiley and Sim (2011), Kollmann et al. (2011), Meh and Moran (2010), Williamson (2012), and Van den Heuvel (2008). The reason why banks exist in my model is purely technological: without banks, the world would be autarchic and agents would be unable to transfer resources across each other and over time. As in the recent work by Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), I give a prominent role to banks by assuming that intermediaries face a balance sheet constraint when obtaining deposits. In these papers however, the shock that causes a financial business cycle is a shock to the quality of bank capital that is, by design, calibrated to produce a downturn as big as in the data. Instead, I either calibrate – in the basic model – the size of the shock by using information on losses suffered by financial intermediaries during the Great Recession, or estimate – in the extended model – all the shocks using Bayesian techniques. The advantage of the estimation strategy is obvious, and opens the avenue for a richer treatment of many of the questions that are left unanswered in the paper. Another important difference is that I combine in the model two sets of financial frictions: on the one hand, banks face frictions in obtaining funds from households; on the other, entrepreneurs face frictions in obtaining funds from banks.

Section 2 describes the basic model and considers how a financial shock that hits the balance sheet of the bank can lead to a decline in output and credit and to a rise in interest rate spreads. Section 3 presents the extended model that is taken to the data and describes the estimation results. Section 4 illustrates the transmission mechanism of financial shocks in the estimated model. Section 5 concludes. Appendices A–D contain additional details on the models and on the data.

## 2. The basic model and the impact of a financial shock

### 2.1. Overview of the model

I consider a discrete-time economy. The economy features three agents: households, bankers, and entrepreneurs. Each agent has a unit mass.<sup>4</sup> Households work, consume and buy real estate, and make one-period deposits into a bank. The household sector in the aggregate is net saver. Entrepreneurs accumulate real estate, hire households, and borrow from banks. In between the households and the entrepreneurs, bankers intermediate funds. The nature of the banking activity implies that bankers are borrowers when it comes to their relationship with households, and are lenders when it comes to their relationship with the credit-dependent sector – the entrepreneurs. I design preferences in a way that two frictions coexist and interact in the model’s equilibrium: first, bankers are credit constrained in how much they can borrow from the patient savers; second, entrepreneurs are credit constrained in how much they can borrow from bankers.

<sup>2</sup> My approach is inspired by a large body of literature, including the recent work by Jermann and Quadrini (2012) who construct time series for financial and technology shocks using a Solow-residual-style approach and show that the series constructed using this approach are highly correlated with those obtained through a Bayesian estimation exercise.

<sup>3</sup> Regarding the focus on estimation, closely related to my work are the papers of Jermann and Quadrini (2012) and Christiano et al. (2014), but these models do not have an explicit modeling of the banking sector.

<sup>4</sup> Except for the introduction of the banking sector, the model structure closely follows a flexible price version of the basic model in Iacoviello (2005), where credit-constrained entrepreneurs borrow from households directly. Here, banks intermediate between households and entrepreneurs.

## 2.2. Main model features

Below, I describe the main features of the model. The complete set of model equations can be found in [Appendix A](#).

### 2.2.1. Households

The representative household chooses consumption  $C_{H,t}$ , housing  $H_{H,t}$ , and time spent working  $N_{H,t}$  to solve the following intertemporal problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta_H^t (\log C_{H,t} + j \log H_{H,t} + \tau \log(1 - N_{H,t})),$$

where  $\beta_H$  is the discount factor, subject to the following flow-of-funds constraint:

$$C_{H,t} + D_t + q_t(H_{H,t} - H_{H,t-1}) = R_{H,t-1}D_{t-1} + W_{H,t}N_{H,t} + \varepsilon_t, \quad (1)$$

where  $D_t$  denotes bank deposits (earning a predetermined, gross return  $R_{H,t}$ ),  $q_t$  is the price of housing in units of consumption, and  $W_{H,t}$  is the wage rate. Housing does not depreciate. The term  $\varepsilon_t$  denotes a redistribution shock that transfers wealth from the bank to the household (the same shock, with opposite sign, appears in the banker's budget constraint too). Here, it captures losses on banks which are gains from the households and, absent equilibrium effects, should wash out in the aggregate (they do not in this model). The optimality conditions yield standard first-order conditions for consumption/deposits, housing demand, and labor supply:

$$\frac{1}{C_{H,t}} = \beta_H E_t \left( \frac{1}{C_{H,t+1}} R_{H,t} \right), \quad (2)$$

$$\frac{q_t}{C_{H,t}} = \frac{j}{H_{H,t}} + \beta_H E_t \left( \frac{q_{t+1}}{C_{H,t+1}} \right), \quad (3)$$

$$\frac{W_{H,t}}{C_{H,t}} = \frac{\tau}{1 - N_{H,t}}. \quad (4)$$

### 2.2.2. Entrepreneurs

The representative entrepreneur solves the following problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta_E^t \log C_{E,t},$$

subject to:

$$C_{E,t} + q_t(H_{E,t} - H_{E,t-1}) + R_{E,t}L_{E,t-1} + W_{H,t}N_{H,t} + aC_{EE,t} = Y_t + L_{E,t}, \quad (5)$$

$$Y_t = H_{E,t-1}^{\nu} N_{H,t}^{1-\nu}, \quad (6)$$

$$L_{E,t} \leq m_H E_t \left( \frac{q_{t+1}}{R_{E,t+1}} H_{E,t} \right) - m_N W_{H,t} N_{H,t}. \quad (7)$$

where Eqs. (5), (6) and (7) denote the budget constraint, the production function and the borrowing constraint respectively.

In Eq. (5), entrepreneurs consume  $C_{E,t}$ , accumulate housing (commercial real estate)  $H_{E,t}$ , produce  $Y_t$  and pay wages to households. The term  $L_{E,t}$  denotes the loans that banks extend to entrepreneurs, yielding a gross return  $R_{E,t}$ . The term  $aC_{EE,t} = \frac{\phi_{EE}}{2} \frac{(L_{E,t} - L_{E,t-1})^2}{L_E}$  (where  $L_E$  denotes the steady-state value of  $L_{E,t}$ ) is a quadratic loan portfolio adjustment cost, assumed to be external to the entrepreneur. This cost penalizes entrepreneurs for changing their loan balances too quickly between one period and the next, and captures the idea that the volume of lending changes slowly over time.<sup>5</sup> Eq. (6) states that real estate, combined with household labor, produces the final output  $Y_t$ .

Eq. (7) is the borrowing constraint. Entrepreneurs cannot borrow more than a fraction  $m_H$  of the expected value of their real estate stock. In addition, the borrowing constraint stipulates that a fraction  $m_N$  of the wage bill must be paid in advance, as in [Neumeyer and Perri \(2005\)](#). I assume that entrepreneurs discount the future more heavily than households and bankers. Formally, their discount factor satisfies the restriction that  $\beta_E < \frac{1}{\gamma_E \frac{1}{\beta_H} + (1-\gamma_E) \frac{1}{\beta_B}}$ . This assumption guarantees that the borrowing constraint will bind in a neighborhood of the steady state.

<sup>5</sup> [Aliaga-Díaz and Olivero \(2010\)](#) present a DSGE model of hold-up effects where switching banks is costly for entrepreneurs. [Curdia and Woodford \(2010\)](#) and [Goodfriend and McCallum \(2007\)](#) develop models of financial intermediation with convex portfolio adjustment costs which mimic the functional form adopted here.

Denote with  $\lambda_{E,t}$  the multiplier associated with the borrowing constraint normalized by the marginal utility of consumption. The optimization conditions for loans, real estate and labor are respectively:

$$\left(1 - \lambda_{E,t} - \frac{\partial ac_{LE,t}}{\partial L_{E,t}}\right) \frac{1}{C_{E,t}} = \beta_E E_t \left(R_{E,t+1} \frac{1}{C_{E,t+1}}\right), \quad (8)$$

$$\left(q_t - \lambda_{E,t} m_H E_t \left(\frac{q_{t+1}}{R_{E,t+1}}\right)\right) \frac{1}{C_{E,t}} = \beta_E E_t \left(\left(q_{t+1} + \frac{vY_{t+1}}{H_{E,t}}\right) \frac{1}{C_{E,t+1}}\right), \quad (9)$$

$$\frac{(1-v)Y_t}{1 + m_N \lambda_{E,t}} = W_{H,t} N_{H,t}. \quad (10)$$

As the first-order conditions show, credit constraints – as measured by the multiplier on the borrowing constraint  $\lambda_{E,t}$  – introduce a wedge between the cost of factors and their marginal product, thus acting as a tax on the demand for credit and the demand for the factors of production. The wedge is intertemporal in the consumption Euler equation (8) and in the real estate demand equation (9); it is intratemporal in the case of the labor demand equation (10).

### 2.2.3. Bankers

The representative banker solves the following problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta_B^t \log C_{B,t}$$

where  $\beta_B < \beta_H$ , subject to:

$$C_{B,t} + R_{H,t-1} D_{t-1} + L_{E,t} + ac_{EB,t} = D_t + R_{E,t} L_{E,t-1} - \varepsilon_t, \quad (11)$$

where  $D_t$  denotes household deposits,  $L_{E,t}$  are loans to entrepreneurs, and  $C_{B,t}$  is banker's private consumption. Note that this formulation is equivalent to a formulation where bankers maximize a convex function of dividends (discounted at rate  $\beta_B$ ), once  $C_{B,t}$  is reinterpreted as the residual income of the banker after depositors have been repaid and loans have been issued. As for the entrepreneurial problem, the term  $ac_{EB,t} = \frac{\phi_{EB}}{2} \frac{(L_{E,t} - L_{E,t-1})^2}{L_E}$  is a quadratic portfolio loan adjustment cost, assumed to be external to the banker. The term  $\varepsilon_t$  is the redistribution shock that, when positive, transfers resources from the bank to the household.

Adjustment cost aside, the flow of funds constraint of the banker implicitly assumes that deposits can be freely converted into loans. To make matters more interesting and more realistic, I assume that the bank is constrained in its ability to issue liabilities by the amount of equity capital (assets less liabilities) in its portfolio. This constraint can be motivated by standard limited commitment problems or by regulatory concerns. For instance, typical regulatory requirements – such as those agreed by the Basel Committee on Banking Supervision – posit that banks hold a capital to assets ratio greater than or equal to some predetermined ratio. Denoting with  $K_{B,t} = L_{E,t} - D_t - E_t \varepsilon_{t+1}$  the bank capital at the beginning of the period (before loan losses caused by redistribution shocks have been realized), a capital adequacy constraint can be reinterpreted as a standard borrowing constraint, that is<sup>6</sup>:

$$D_t \leq \gamma_E (L_{E,t} - E_t \varepsilon_{t+1}). \quad (12)$$

In Eq. (12), the left-hand side denotes banks liabilities  $D_t$ , while the right-hand side denotes the fraction of bank assets that can be used as collateral, once expected losses are taken into account.

Let  $m_{B,t} \equiv \beta_B E_t (C_{B,t} / C_{B,t+1})$  denote the banker's stochastic discount factor. Denote with  $\lambda_{B,t}$  the multiplier on the capital adequacy constraint normalized by the marginal utility of consumption. The optimality conditions for deposits and loans are respectively:

$$1 - \lambda_{B,t} = E_t (m_{B,t} R_{H,t}), \quad (13)$$

$$1 - \gamma_E \lambda_{B,t} + \frac{\partial ac_{EB,t}}{\partial L_{E,t}} = E_t (m_{B,t} R_{E,t+1}). \quad (14)$$

The interpretation of the two first-order condition is straightforward. It also illustrates why deposits  $D_t$  and loans  $L_{E,t}$  pay different returns in equilibrium. Consider the ways a bank can increase its consumption by one extra unit today:

1. The banker can consume more today by borrowing from the household, increasing deposits  $D_t$  by one unit. By doing so, the bank reduces its equity by one unit, thus tightening its borrowing constraint one-for-one and reducing the utility value of an extra deposit by  $\lambda_{B,t}$ . Overall, today's payoff from the deposit is  $1 - \lambda_{B,t}$ . The next-period expected cost is given by the stochastic discount factor times the interest rate  $R_{H,t}$ .

<sup>6</sup> For the extended model, Appendix B derives the borrowing constraint starting from the capital adequacy constraint.

2. The banker can consume more today by reducing loans by one unit. By lending less, the bank tightens its borrowing constraint, since it reduces its equity. The utility cost of tightening the borrowing constraint through lower loans is equal to  $\gamma_E \lambda_{B,t}$ . Intuitively, the more loans are useful as collateral for the bank activity (the higher  $\gamma_E$  is), the larger is the utility cost of reducing loans.

For the bank to be indifferent between collecting deposits and making loans, the adjusted returns across loans and deposits must be equalized. Given that  $R_{H,t}$  is determined from the household problem, the banker will be borrowing constrained, and  $\lambda_{B,t}$  will be positive, if  $m_{B,t}$  is sufficiently lower than the inverse of  $R_{H,t}$ . In turn, if  $\lambda_{B,t}$  is positive, the required returns on loans  $R_{E,t}$  will be higher, the lower  $\gamma_E$  is. Intuitively, when  $\gamma_E$  is low, the liquidity value of loans is lower, and the compensation required by the bank to be indifferent between lending and borrowing becomes higher. Moreover, loans will pay a return that is (near the steady state) higher than the cost of deposits, since, so long as  $\gamma_E$  is lower than one, they are less liquid than deposits.

#### 2.2.4. Market clearing

I normalize the total supply of housing to unity. The market clearing conditions for goods and housing are respectively:

$$Y_t = C_{H,t} + C_{B,t} + C_{E,t}, \quad (15)$$

$$H_{E,t} + H_{H,t} = 1. \quad (16)$$

#### 2.2.5. Steady state properties

In the non-stochastic steady state of the model, the interest rate on deposits equals the inverse of the household discount factor. This can be seen immediately from Eq. (2) evaluated at steady state. That is:

$$R_H = \frac{1}{\beta_H}. \quad (17)$$

In addition, when evaluated at their non-stochastic steady state, Eqs. (13) and (14) imply that: (1) so long as  $\beta_B < \beta_H$  (bankers are impatient), the bankers will be credit constrained and; (2) so long as  $\gamma_E$  is smaller than one, there will be a positive spread between the return on loans and the cost of deposits. The spread will increase with the tightness of the capital requirement constraint for the bank. Formally:

$$\lambda_B = 1 - \beta_B R_H = 1 - \frac{\beta_B}{\beta_H} > 0, \quad (18)$$

$$R_E = \frac{1}{\beta_B} - \gamma_E \left( \frac{1}{\beta_B} - \frac{1}{\beta_H} \right) > R_H. \quad (19)$$

I turn now to entrepreneurs. Given the interest rates on loans  $R_E$ , a necessary condition for entrepreneurs to be constrained is that their discount factor is lower than the inverse of the return on loans above. When this condition is satisfied (that is,  $\beta_E R_E < 1$ ), entrepreneurs will be constrained in a neighborhood of the steady state. Alternatively, this condition requires that the entrepreneurial discount rate is higher than a weighted average of the discount rates of households and bankers:

$$\frac{1}{\beta_E} > \gamma_E \frac{1}{\beta_H} + (1 - \gamma_E) \frac{1}{\beta_B}. \quad (20)$$

Both the bankers' credit constraint and the entrepreneurs' credit constraint create a positive wedge between the steady-state output in absence of financial frictions and the output when financial frictions are present. The credit constraint on banks limits the amount of savings that banks can transform into loans. Likewise, the credit constraint on entrepreneurs limits the amount of loans that can be invested for production. Both constraints lead to lower steady-state output. The same forces are also at work for shocks that move the economy away from the steady state, to the extent that these shocks tighten or loosen the severity of the borrowing constraints.

### 2.3. Calibration

To illustrate the main workings of the model, I study the macroeconomic consequences of a shock that persistently reduces bank equity. In the full estimated model, I will also look at other shocks, and estimate using Bayesian methods the model's structural parameters. The parameters chosen here are in line with the estimates and the calibration of the extended model.

The time period is a quarter. I set the discount factors of households, entrepreneurs and bankers respectively at  $\beta_H = 0.9925$ ,  $\beta_E = 0.94$  and  $\beta_B = 0.945$ . Together with the choice of the leverage parameters (described below), these numbers imply an annualized steady-state deposit rate  $R_H$  of 3 percent and a steady-state lending rate  $R_E$  of 5 percent. As for adjustment cost parameters for loans I set both  $\phi_{EE}$  and  $\phi_{EB}$  equal to 0.25.

I set the weight on leisure in the household utility function,  $\tau$ , at 2, implying a share of active time spent working close to one half, and a Frisch labor supply elasticity around 1. I set the share of housing in production  $\nu$  is set at 0.05, and the preference parameter for housing  $j$  in the utility function at 0.075. These choices imply a ratio of real estate wealth to output of 3.1 (annualized), of which 0.8 is commercial real estate and 2.3 is residential real estate.

I next choose the parameters controlling leverage. I set  $m_N = 1$ , so that all labor costs must be paid in advance. I set  $m_H$ , the entrepreneurial loan-to-value (LTV) ratio, to 0.9. The leverage parameter for the bank is set at  $\gamma_E = 0.9$ : this number is consistent with historical data on bank balance sheets that show capital-asset ratios for banks close to 0.1 (see for instance the evidence in Van den Heuvel, 2008).

#### 2.4. The dynamic effects of a financial shock

To gain intuition into the workings of the model, it is useful to consider how time-variation in the tightness of the bankers' borrowing constraint can affect equilibrium dynamics.

I begin with the price side. Abstracting from adjustment costs, the expression for the spread between the return on loans and the cost of deposits can be written as:

$$E_t(R_{E,t+1}) - R_{H,t} = \frac{\lambda_{B,t}}{m_{B,t}}(1 - \gamma_E). \quad (21)$$

According to this expression, the spread between the return on entrepreneurial loans and the cost of deposits becomes larger whenever the banker's multiplier on the borrowing constraint  $\lambda_{B,t}$  gets higher. When the capital adequacy constraint becomes tighter, for instance because bank net worth is lower, the bank requires a larger return on its assets in order to be indifferent between extending loans and issuing deposits. This occurs because loans are more illiquid than deposits: when the constraint is binding, a decline in deposits of 1 dollar requires a decline in loans by  $\frac{1}{\gamma_E} > 1$  dollars. Accordingly, the rise in the spread will act as a drag on economic activity during periods of lower bank net worth.

I move now to the quantity side. Whenever a shock causes a reduction in bank capital, the logic of the balance sheet requires the bank to contract its assets by a multiple of its capital, in order for the bank to restore its leverage ratio. The banker could avoid this by raising new capital or by reducing consumption. However, the bankers' impatience makes this route impractical as well as insufficient. As a consequence, the bank reduces its lending. If the productive sector of the economy depends on bank credit to run its activities, the contraction in bank credit causes in turn a recession.

How much do financial shocks affect the economy? Here I consider the effect of the shock  $\varepsilon_t$  that transfers resources from the bank to the household. An interpretation of this shock is that it captures losses for the banking system caused, for instance, by a wave of loan defaults. Granted, loan defaults are not exogenous events, and they may have broader consequences than just hitting the balance sheet of lenders, for at least two reasons. First, there are large legal and social costs associated with defaults. Second, defaults are naturally the symptom of some primitive economic distress for those who default, which ideally one would like the capture in a richer model. With these caveats in mind, I size the redistribution shocks by looking at the data on loan losses – caused directly or indirectly by defaults.

The particular type of shock that I emphasize here only captures one of the ways in which episodes of financial stress may ultimately redistribute resources across agents. In addition, both in the basic model of this section and in the estimated model of the next section, I place emphasis on a shock that redistributes wealth away from the banks towards the household sector. This is in keeping with the observation that the large losses suffered from banks during the Great Recession originated from household defaults. In the basic model presented here, there is only one household type (the savers), so that household-savers gain. In the extended model of the next section, which includes both household-savers and household-borrowers, I assume that household-borrowers gain. For aggregate dynamics, whether the gains accrue to household who save or households to borrow is not crucial: what matters is that wealth gets redistributed away from a relatively productive sector (the banking sector that lends to entrepreneurs) to a relatively unproductive one.

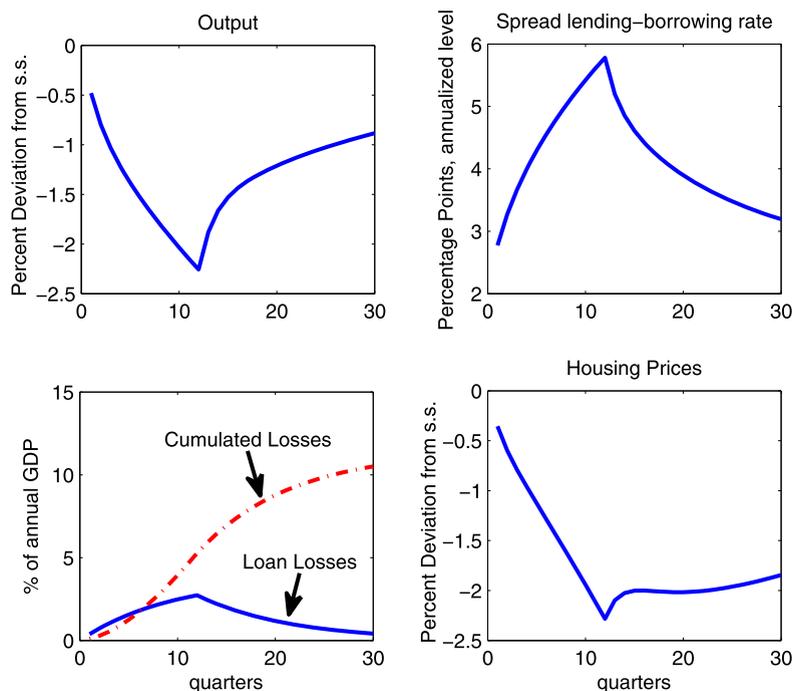
Fig. 1 plots a dynamic simulation for the model economy in response to a sequence of redistribution shocks that hit the balance sheet of the bank. I assume that the stochastic process for  $\varepsilon_t$  follows

$$\varepsilon_t = 0.9\varepsilon_{t-1} + \iota_t. \quad (22)$$

I feed into the model a sequence of unexpected shocks to  $\iota_t$ , each quarter equal to 0.38 percent of annual GDP, which lasts 12 quarters and causes losses for the banking system to rise from zero to 2.8 percent of GDP after 3 years, before loan losses gradually return to zero.<sup>7</sup> Note that the losses for the banking system are equal to the gains of household sector, so no wealth is created or destroyed in aggregate by the shock. As such, the shock is a pure redistribution shock.

From the standpoint of the banks, the loan losses closely mimic the losses of financial system during the Great Recession. Between 2007Q1 and 2009Q4, annualized loan charge-off rates on residential mortgages rose from 0.1 percent to 2.8 percent,

<sup>7</sup> In the experiment reported here, the cumulative loan losses for banks are about 9 percent of annual GDP after 5 years. These numbers are in the ballpark of the IMF estimates of total writedowns by banks and other financial institutions which were made during the financial crisis. See for instance Table 1.3 in IMF (2009).



**Fig. 1.** Dynamics of the basic model after default shock. Note: The plots show the responses of macroeconomic variables to a shock that leads after 3 years to (flow) loan losses for banks equal to 2.8 percent of GDP. The cumulated losses are the cumulative sum of the flow loan losses, divided by 4 to express as a fraction of annual GDP.

and charge-off rates on consumer loans rose from 2.7 percent to 6.6 percent. Given a ratio of total household debt to GDP close to 1, the shock here mimics the increase in loan charge-offs of the Great Recession. Note also that throughout the paper, my maintained assumption is that banks cannot react to the shock by charging higher interest rates.

The shock impairs the bank's balance sheet, by reducing the value of the banks' assets (total loans minus loan losses) relative to the liabilities (household deposits). Given the shock, in absence of any further adjustment to either loans or deposits, the bank would have a capital–asset ratio that is below target. The bank could restore such ratio either by deleveraging (reducing deposits from households), or by reducing consumption in order to restore its equity cushion. If reducing consumption is costly, the bank cuts back on its loans, and begins a vicious, dynamic circle of simultaneous reduction both in loans and deposits, thus propagating the credit crunch. In particular, the decline in loans to the credit-dependent sector of the economy (entrepreneurs) acts a drag on both consumption and productive investment. It drags investment down because credit-constrained entrepreneurs reduce their real estate holdings and labor demand as credit supply is reduced. And it drags consumption down because the decline in labor demand and the reduction in entrepreneurial investment induce a decline in total output.<sup>8</sup> All told, the shock produces a large and persistent decline in economic activity. After 3 years, output and asset prices are more than 2 percent below baseline, and the spread between lending and deposit rates, which equals 2 percent in steady state, rises to almost 6 percent.

### 3. Extended model and structural estimation

#### 3.1. Overview of the model

The basic model of the previous section assumes that real estate is the only input in production, that there is no heterogeneity across households, and that all the productive assets in the economy are held by firms that are credit constrained. In addition, the model lacks a horse race between “financial” shocks and other shocks that could be potentially important for explaining business fluctuations. In this section, I extend the basic model by relaxing the assumptions above. I then take the model to the data using likelihood-based techniques. An advantage of this approach is that the estimation provides an in-sample accounting of the forces driving recent U.S. business cycles in general, and the Great Recession in particular.

Relative to the model of the previous section, I split the household sector into two types. Alongside patient households, there is a group of impatient households that earns a fraction  $\sigma$  of the total wage income in the economy and borrows against their homes. In addition, patient households accumulate a share  $1 - \mu$  of the economywide capital stock, while

<sup>8</sup> An additional force that reduces output in the wake of a redistribution shock is a negative wealth effect on labor supply for the households who receive funds from the bank. This effect contributes to less than one quarter of the decline in output.

entrepreneurs accumulate real estate (as before) and the remaining fraction  $\mu$  of the capital stock. Banks collect deposits and make loans to either impatient households or entrepreneurs. To capture the slow dynamics of many macroeconomic variables, I allow for quadratic adjustment costs for all assets, for habits in consumption, and for inertia in the borrowing constraints and in the capital adequacy constraint. With appropriate choices of the parameters, the model nests either the basic model of the previous section or the standard RBC model as special cases. Finally, as in virtually every model that is estimated using likelihood-based techniques, I allow for a rich array of shocks to explain the variation in the data.

### 3.2. Main model features

Below, I describe the main features of the model. The complete set of model equations can be found in [Appendix B](#).

#### 3.2.1. Patient households

The patient households objective is given by

$$\max E_0 \sum_{t=0}^{\infty} \beta_H^t (A_{p,t}(1-\eta) \log(C_{H,t} - \eta C_{H,t-1}) + j A_{j,t} A_{p,t} \log H_{H,t} + \tau \log(1 - N_{H,t})),$$

subject to the following budget constraint:

$$\begin{aligned} C_{H,t} + \frac{K_{H,t}}{A_{K,t}} + D_t + q_t(H_{H,t} - H_{H,t-1}) + ac_{KH,t} + ac_{DH,t} \\ = \left( R_{M,t} z_{KH,t} + \frac{1 - \delta_{KH,t}}{A_{K,t}} \right) K_{H,t-1} + R_{H,t-1} D_{t-1} + W_{H,t} N_{H,t}. \end{aligned} \quad (23)$$

In the utility function above, the term  $A_{p,t}$  denotes a shock to preferences for consumption and housing jointly (aggregate spending shock), the  $A_{j,t}$  term denotes a housing demand shock, and  $\eta$  measures external habits in consumption. In the budget constraint, households own physical capital  $K_{H,t}$  and rent capital services  $z_{KH,t} K_{H,t}$  to entrepreneurs at the rental rate  $R_{M,t}$  (the utilization rate is  $z_{KH,t}$ ). The term  $A_{K,t}$  denotes an investment-specific technology shock. The terms  $ac_{KH,t}$  and  $ac_{DH,t}$  denote convex, external adjustment costs for capital and deposits. The parameter  $\delta_{KH,t}$  denotes the depreciation function for physical capital, which assumes that depreciation is convex in the utilization rate of capital. The functional forms for the adjustment costs and for the depreciation function are in [Appendix B](#).

#### 3.2.2. Impatient households

The objective of the impatient households is given by

$$\max E_0 \sum_{t=0}^{\infty} \beta_S^t (A_{p,t}(1-\eta) \log(C_{S,t} - \eta C_{S,t-1}) + j A_{j,t} A_{p,t} \log H_{S,t} + \tau \log(1 - N_{S,t})),$$

where  $\beta_S$  denotes their discount factor.<sup>9</sup> Their budget constraint is:

$$C_{S,t} + q_t(H_{S,t} - H_{S,t-1}) + R_{S,t-1} L_{S,t-1} - \varepsilon_{H,t} + ac_{SS,t} = L_{S,t} + W_{S,t} N_{S,t}, \quad (24)$$

where  $L_{S,t}$  denotes loans made by banks to impatient households, paying a gross interest rate  $R_{S,t}$ , and the term  $ac_{SS,t}$  denotes a convex cost of adjusting loans from one period to the next. The term  $\varepsilon_{H,t}$  in the budget constraint is an exogenous shock, similar to the redistribution shock of the previous section, that transfers wealth from banks to households: I assume that impatient households can pay back less (more) than agreed on their contractual obligations when  $\varepsilon_{H,t}$  is greater (smaller) than zero. From the households' perspective, this redistribution shock represents – all else equal – a positive shock to wealth, since it allows them to spend more than previously anticipated. When I take the model to the data, I measure this shock by looking at data on loan losses on residential mortgages suffered by financial intermediaries.

Impatient households are also subject to a borrowing constraint that limits their liabilities to a fraction of the value of their house:

$$L_{S,t} \leq \rho_S L_{S,t-1} + (1 - \rho_S) m_S A_{MH,t} E_t \left( \frac{q_{t+1}}{R_{S,t}} H_{S,t} \right). \quad (25)$$

The term  $\rho_S$  allows for slow adjustment over time of the borrowing constraint, to capture the idea that in practice lenders do not readjust borrowing limits every quarter. The term  $A_{MH,t}$  denotes an exogenous shock to the borrowing capacity of the household, due to, for instance, looser screening practices of the banks that allow them to supply more loans for given amount of collateral. The borrowing constraint binds in a neighborhood of the steady state if  $\beta_S$  is lower than a weighted average of the discount factors of patient households and bankers.

<sup>9</sup> For impatient households to borrow and to be credit constrained in equilibrium, one needs to assume that their discount factor is lower than a weighted average of the discount factors of households and banks. See [Appendix B](#) for details. An analogous restriction applies to entrepreneurs.

Note that one could endogenize the default-redistribution shock in other ways: for instance, one could assume that if house prices fall below some value, borrowers could find it optimal to default rather than roll their debt over: defaulting would then be equivalent to choosing a value for  $R_{S,t}L_{S,t-1}$  lower than previously agreed.

### 3.2.3. Bankers

Bankers solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta_B^t (1 - \eta) \log(C_{B,t} - \eta C_{B,t-1})$$

subject to the following budget constraint:

$$\begin{aligned} C_{B,t} + R_{H,t-1}D_{t-1} + L_{E,t} + L_{S,t} + ac_{DB,t} + ac_{EB,t} + ac_{SB,t} \\ = D_t + R_{E,t}L_{E,t-1} + R_{S,t}L_{S,t-1} - \varepsilon_{E,t} - \varepsilon_{H,t}. \end{aligned} \quad (26)$$

The last two terms denote the repayment shocks. As before, the terms  $ac_{DB,t}$ ,  $ac_{EB,t}$  and  $ac_{SB,t}$  denote adjustment costs paid by the bank for adjusting deposits, loans to entrepreneurs  $L_{E,t}$ , and loans to impatient households  $L_{S,t}$ . The bank is subject to a capital adequacy constraint of the form:

$$L_t - D_t - E_t \varepsilon_{t+1} \geq \rho_D (L_{t-1} - D_{t-1} - E_{t-1} \varepsilon_t) + (1 - \gamma)(1 - \rho_D)(L_t - E_t \varepsilon_{t+1}), \quad (27)$$

where  $L_t = L_{E,t} + L_{S,t}$  are bank loans and  $\varepsilon_t = \varepsilon_{E,t} + \varepsilon_{H,t}$  are loan losses. This constraint posits that bank equity (after expected losses) must exceed a fraction of bank assets, allowing for partial adjustment in bank capital given by  $\rho_D$ . In this formulation, the capital–asset ratio of the bank can temporarily deviate from its long-run target,  $\gamma$ , so long as  $\rho_D$  is not equal to zero. Such a formulation allows the bank to take corrective action to restore its capital–asset ratio beyond one period.

### 3.2.4. Entrepreneurs

The last group of agents are the entrepreneurs. They hire workers and combine them with capital (both produced by them and supplied by patient households) in order to produce the final good  $Y_t$ . Their utility function is

$$\max E_0 \sum_{t=0}^{\infty} \beta_E^t (1 - \eta) \log(C_{E,t} - \eta C_{E,t-1})$$

and they are subject to the following budget constraint:

$$\begin{aligned} C_{E,t} + \frac{K_{E,t}}{A_{K,t}} + q_t H_{E,t} + R_{E,t}L_{E,t-1} + W_{H,t}N_{H,t} + W_{S,t}N_{S,t} + R_{M,t}Z_{KH,t}K_{H,t-1} + ac_{KE,t} + ac_{EE,t} \\ = Y_t + \frac{1 - \delta_{KE,t}}{A_{K,t}} K_{E,t-1} + q_t H_{E,t-1} + L_{E,t} + \varepsilon_{E,t}, \end{aligned} \quad (28)$$

where  $\varepsilon_{E,t}$  denotes default-redistribution shocks, and  $ac_{KE,t}$  and  $ac_{EE,t}$  denote adjustment costs for capital and loans. The production function is given by:

$$Y_t = A_{Z,t} (Z_{KH,t} K_{H,t-1})^{\alpha(1-\mu)} (Z_{KE,t} K_{E,t-1})^{\alpha\mu} H_{E,t-1}^{\nu} N_{H,t}^{(1-\alpha-\nu)(1-\sigma)} N_{S,t}^{(1-\alpha-\nu)\sigma}, \quad (29)$$

where  $A_{Z,t}$  is a shock to total factor productivity. Finally, entrepreneurs are subject to a borrowing constraint that acts as a wedge on the capital and labor demand. The constraint is given by:

$$L_{E,t} \leq \rho_E L_{E,t-1} + (1 - \rho_E) A_{ME,t} \left( m_H E_t \left( \frac{q_{t+1}}{R_{E,t+1}} H_{E,t} \right) + m_K K_{E,t} - m_N (W_{H,t} N_{H,t} + W_{S,t} N_{S,t}) \right). \quad (30)$$

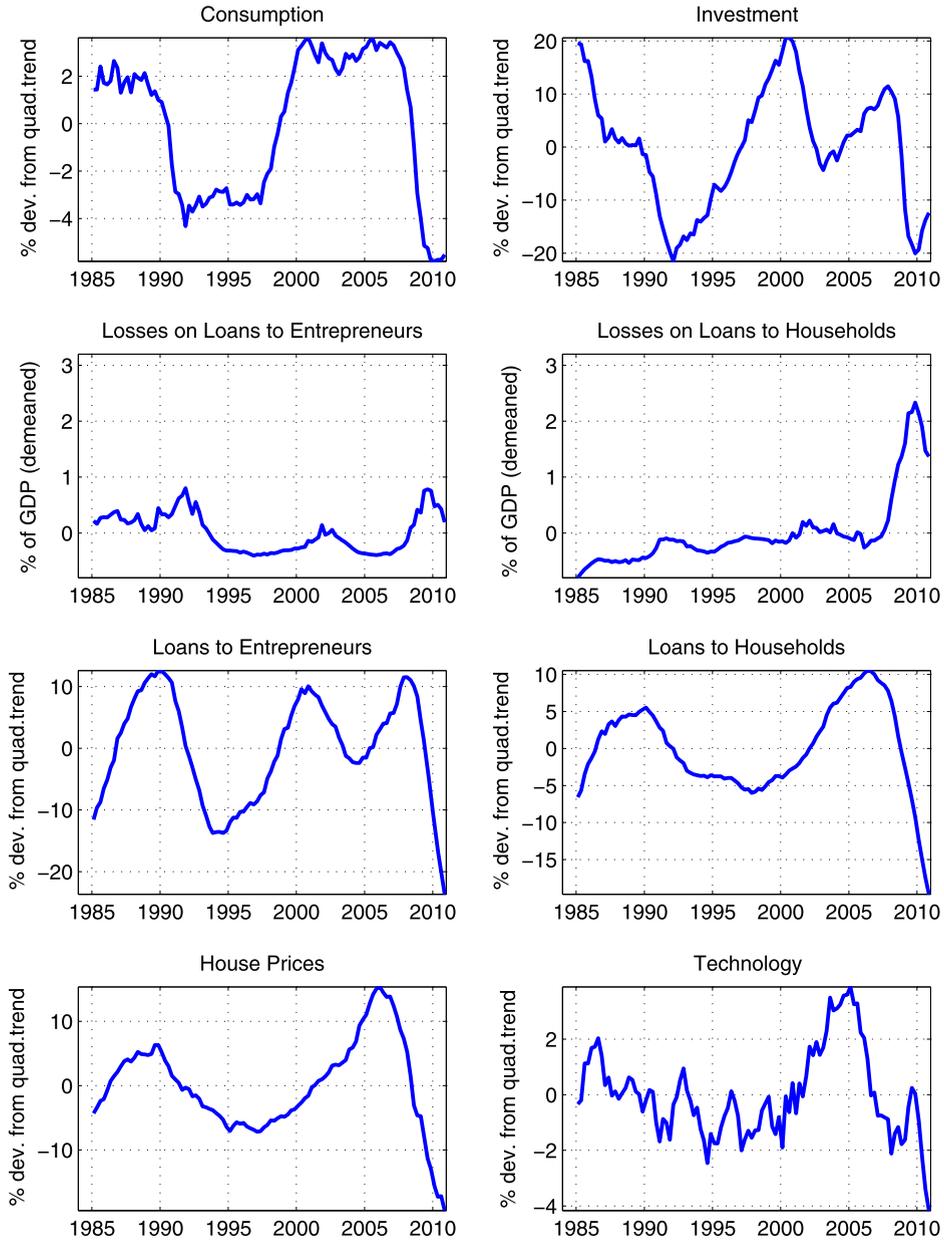
In a manner similar to the impatient households problem, the term  $A_{ME,t}$  denotes a shock to the borrowing capacity of the entrepreneur.

### 3.2.5. Market clearing and equilibrium

Market clearing is implied by Walras's law by aggregating all the budget constraints. For housing, we have the following market clearing condition:

$$H_{H,t} + H_{S,t} + H_{E,t} = 1. \quad (31)$$

To compute the model dynamics, I solve a linearized version of the system of equations describing the equilibrium of the model under the maintained assumption that the constraints given by Eqs. (25), (27) and (30) are always binding. I verify that, given the size of the estimated shocks, the Lagrange multipliers are always positive throughout a given simulation.



**Fig. 2.** Data used in estimation. Note: The model parameters are estimated using data from 1990Q1 to 2010Q4. The 1985–1989 period is used to initialize the Kalman filter.

### 3.3. Estimation

I use Bayesian methods as described in [An and Schorfheide \(2007\)](#) to estimate the model parameters.

#### 3.3.1. Data

The emphasis on financial factors of this paper leads me to consider for estimation several quantities which are important to identify the various shocks given the data. Accordingly, I estimate the model using U.S. quarterly data from 1985Q1 to 2010Q4.<sup>10</sup> The model allows for eight shocks. Following usual practice, I use as many shocks as observable variables. The observables are: real consumption, real nonresidential fixed investment, losses on loans to businesses, losses on loans to households, loans to businesses, loans to households, real house prices, and total factor productivity. [Appendix C](#) describes

<sup>10</sup> The sample begins in 1985Q1, but the first 20 observations are used as a training sample for the Kalman filter, so that the estimation is effectively based on the observations from 1990Q1 to 2010Q4.

**Table 1**  
Calibrated parameters for the extended model.

Parameter		Value
Household-saver (HS) discount factor	$\beta_H$	0.9925
Household-borrower (HB) discount factor	$\beta_S$	0.94
Banker discount factor	$\beta_B$	0.945
Entrepreneur (E) discount factor	$\beta_E$	0.94
Total capital share in production	$\alpha$	0.35
Loan-to-value ratio on housing, HB	$m_S$	0.9
Loan-to-value ratio on housing, E	$m_H$	0.9
Loan-to-value ratio on capital, E	$m_K$	0.9
Wage bill paid in advance	$m_N$	1
Liabilities to assets ratio for Banker	$\gamma_E, \gamma_S$	0.9
Housing preference share	$j$	0.075
Capital depreciation rates	$\delta_{KE}, \delta_{KH}$	0.035
Labor Supply parameter	$\tau$	2

**Table 2a**  
Estimation, structural parameters.

Parameter		Prior distribution			Posterior distribution		
		Density	Mean	St.dev.	5%	Mean	95%
Habit in consumption	$\eta$	beta	<b>0.5</b>	0.15	0.36	<b>0.46</b>	0.56
D adj. cost, Banks	$\phi_{DB}$	gamm	<b>0.25</b>	0.125	0.05	<b>0.14</b>	0.26
D adj. cost, Household Saver (HS)	$\phi_{DH}$	gamm	<b>0.25</b>	0.125	0.04	<b>0.10</b>	0.20
K adj. cost, Entrepreneurs (E)	$\phi_{KE}$	gamm	<b>1</b>	0.5	0.23	<b>0.59</b>	1.41
K adj. cost, Household Saver (HS)	$\phi_{KH}$	gamm	<b>1</b>	0.5	0.88	<b>1.73</b>	2.95
Loan to E adj. cost, Banks	$\phi_{EB}$	gamm	<b>0.25</b>	0.125	0.03	<b>0.07</b>	0.13
Loan to E adj. cost, E	$\phi_{EE}$	gamm	<b>0.25</b>	0.125	0.02	<b>0.06</b>	0.11
Loan to HB adj. cost, Banks	$\phi_{SB}$	gamm	<b>0.25</b>	0.125	0.24	<b>0.47</b>	0.72
Loan to HB adj. cost, HH Borrower HB	$\phi_{SS}$	gamm	<b>0.25</b>	0.125	0.14	<b>0.37</b>	0.66
Capital share of E	$\mu$	beta	<b>0.5</b>	0.1	0.34	<b>0.46</b>	0.58
Housing share of E	$\nu$	beta	<b>0.04</b>	0.01	0.03	<b>0.04</b>	0.05
Inertia in capital adequacy constraint	$\rho_D$	beta	<b>0.25</b>	0.1	0.10	<b>0.24</b>	0.41
Inertia in E borrowing constraint	$\rho_E$	beta	<b>0.25</b>	0.1	0.53	<b>0.65</b>	0.79
Inertia in HB borrowing constraint	$\rho_S$	beta	<b>0.25</b>	0.1	0.64	<b>0.70</b>	0.76
Wage share HB	$\sigma$	beta	<b>0.3</b>	0.1	0.22	<b>0.33</b>	0.45
Curvature for utilization function E	$\zeta_E$	beta	<b>0.2</b>	0.1	0.20	<b>0.42</b>	0.63
Curvature for utilization function HS	$\zeta_H$	beta	<b>0.2</b>	0.1	0.18	<b>0.38</b>	0.58

the data construction. Except for loan losses, I detrend the logarithm of each variable independently using a quadratic trend.<sup>11</sup> The detrended and demeaned data are plotted in Fig. 2.

### 3.3.2. Calibration and priors

Table 1 summarizes the calibrated parameters. These values are kept fixed because the data are demeaned and cannot pin down steady-state values in the estimation procedure. I set the variable capital share in production  $\alpha$  at 0.35 and capital depreciation rate at 0.035. I choose a number for the depreciation rate which is slightly larger than the typical number in the literature – 0.025 – since my model also includes real estate as a factor of production which does not depreciate altogether. These numbers imply an investment to output ratio of 0.25 and a variable capital to output ratio of 1.8. All the leverage parameters are set at 0.9, and I assume labor must be fully paid in advance, so that  $m_N = 1$ . Together with the discount factors, the leverage parameters imply an annualized steady-state return on deposits of 3 percent and a steady-state return on loans of 5 percent.

Tables 2a and 2b show the prior distributions for the model's remaining parameters. I assume that all parameters are independent a priori. The domain of most parameters, whenever possible, covers a wide range of outcomes. In the prior, I choose to be conservative about the importance of financial shocks. In particular, my assumptions about the relative importance of the various shocks imply that, at the prior mean, the financial shocks (that is, the combination of housing price shocks, default-redistribution shocks, and loan-to-value ratio shocks) account for about 15 percent of the total variance of output, consumption and investment at business cycle frequencies (as implied by an HP-filter with a smoothing parameter of 1600).

<sup>11</sup> Although several recent estimated DSGE models allow for deterministic or stochastic trends, incorporating such features into a model with financial variables such as loans is nontrivial. Several financial variables appear to have trends of their own which would require specific modeling assumptions to guarantee balanced growth: for instance, the ratio of household debt to GDP has been rising throughout the sample in question. I leave exploration of this topic for future research.

**Table 2b**  
Estimation, shock processes.

Parameter		Prior distribution			Posterior distribution		
		Density	Mean	St.dev.	5%	Mean	95%
Autocor. E default shock	$\rho_{be}$	beta	<b>0.8</b>	0.1	0.886	<b>0.932</b>	0.971
Autocor. HB default shock	$\rho_{bh}$	beta	<b>0.8</b>	0.1	0.944	<b>0.969</b>	0.988
Autocor. housing demand shock	$\rho_j$	beta	<b>0.8</b>	0.1	0.986	<b>0.992</b>	0.997
Autocor. investment shock	$\rho_k$	beta	<b>0.8</b>	0.1	0.840	<b>0.916</b>	0.973
Autocor. LTV shock, E	$\rho_{me}$	beta	<b>0.8</b>	0.1	0.750	<b>0.839</b>	0.917
Autocor. LTV shock, HB	$\rho_{mh}$	beta	<b>0.8</b>	0.1	0.781	<b>0.873</b>	0.948
Autocor. preference shock	$\rho_p$	beta	<b>0.8</b>	0.1	0.989	<b>0.994</b>	0.998
Autocor. technology shock	$\rho_z$	beta	<b>0.8</b>	0.1	0.973	<b>0.988</b>	0.997
St.dev., default shock, E	$\sigma_{be}$	invg	<b>0.0025</b>	0.025	0.0009	<b>0.0011</b>	0.0012
St.dev., default shock, HB	$\sigma_{bh}$	invg	<b>0.0025</b>	0.025	0.0012	<b>0.0013</b>	0.0015
St.dev., housing demand shock	$\sigma_j$	invg	<b>0.05</b>	0.05	0.0248	<b>0.0346</b>	0.0473
St.dev., investment shock	$\sigma_k$	invg	<b>0.005</b>	0.025	0.0049	<b>0.0081</b>	0.0161
St.dev., LTV shock, E	$\sigma_{me}$	invg	<b>0.0025</b>	0.025	0.0129	<b>0.0204</b>	0.0366
St.dev., LTV shock, HB	$\sigma_{mh}$	invg	<b>0.0025</b>	0.025	0.0090	<b>0.0115</b>	0.0150
St.dev., preference shock	$\sigma_p$	invg	<b>0.005</b>	0.025	0.0179	<b>0.0205</b>	0.0237
St.dev., technology shock	$\sigma_z$	invg	<b>0.005</b>	0.025	0.0062	<b>0.0070</b>	0.0080

Note: The posterior density is constructed by simulation using the Random-Walk Metropolis algorithm (with 250,000 draws) as described in [An and Schorfheide \(2007\)](#).

### 3.3.3. Estimation findings

The last three columns of [Tables 2a and 2b](#) report the means and 5% and 95% of the posterior distribution for the estimated model parameters. All shocks are estimated to be quite persistent, with autocorrelation coefficients ranging from 0.84 to 0.994. The share of constrained entrepreneurs,  $\mu$ , is found to be 0.46, slightly lower than its 0.5 prior. The wage share of constrained households,  $\sigma$ , is found to be 0.33, slightly higher than its 0.3 prior. The elasticity of output to entrepreneurial real estate ( $\nu$ ) is estimated at 0.04, implying a steady-state ratio of commercial real estate to annual output of about 0.4.

I find substantially more inertia in the household and entrepreneurs' borrowing constraints (around 0.7) than in the capital adequacy constraint of the bank. Interestingly, the inertia in the borrowing constraints lines up with the well-known observation that various indicators of the quantity of credit tend to lag the business cycle, rather than lead it.

The estimated standard deviation of the household default shock is only 0.13 percentage points. Seen through the lenses of the model, the experience of the financial crisis, when charge-offs rates on loans to households rose by more than 2 percentage points (see [Fig. 2](#)), appears a remarkably rare event.

## 4. The transmission of financial shocks

### 4.1. Financial shocks and the great recession

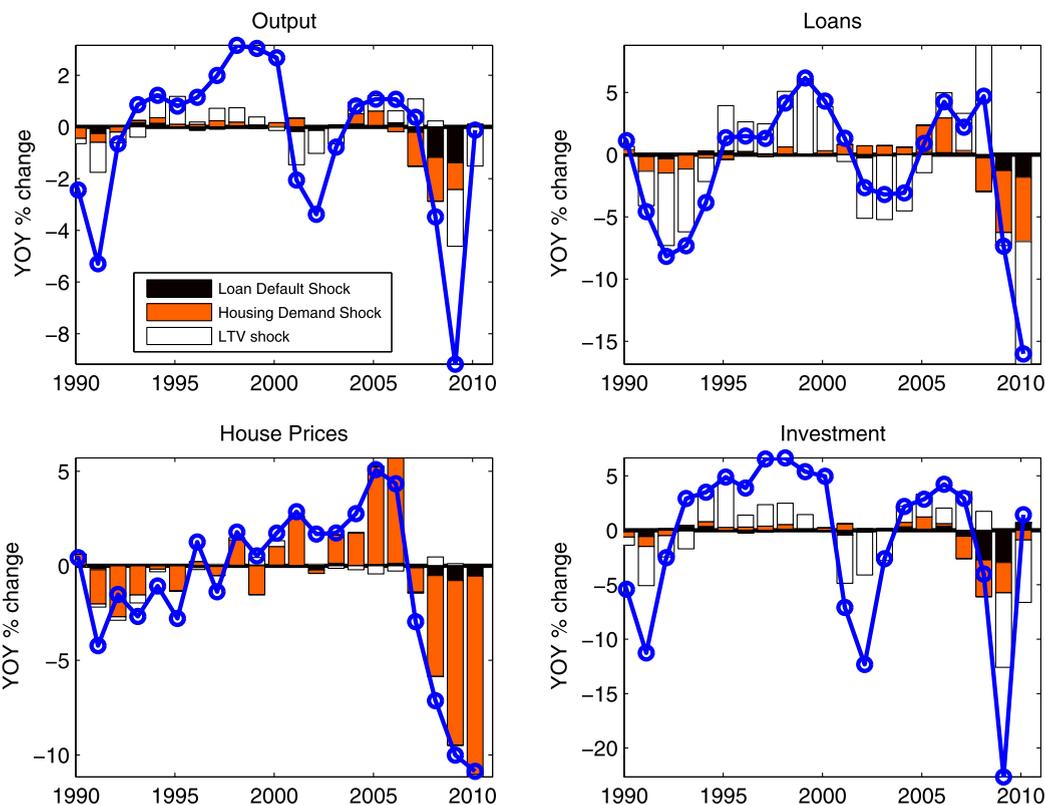
An important question that one can ask of the estimated model is: how important were financial shocks in shaping the recent U.S. macroeconomic experience? [Fig. 3](#) provides an answer by providing historical decompositions of output, total loans, house prices and investment over the estimation sample (at the mean of the estimated parameters). In the data – consistent with the model – output is defined as the sum of total consumption and nonresidential fixed investment, thus excluding the foreign and the government sector. As the figure shows, movements in output and investment do not appear to be driven much by financial shocks until 2007, but the Great Recession offers a remarkably different picture, as also shown in [Table 3](#). During the Great Recession, about two-thirds of the decline in output and investment is driven by the combined effect of default shocks, housing demand shocks, and LTV shocks. The timing of the shocks, in particular, is of independent interest. Early during the Recession in 2007 and 2008, the decline in output and investment is mostly driven by negative housing demand shocks. Lower collateral values reduce the borrowing capacity of entrepreneurs and lead to lower investment and output. Next, default shocks take center stage. Default shocks account for 1.2 percentage points of the 3.6 percent decline in output in 2008, and for 1.4 percentage points of the 9 percent decline in output in 2009. Last, LTV shocks become important. In 2010, with output growth nearly recovering, tighter credit – in the form of negative LTV shocks – subtracts 1.5 percent from output growth. All told, the three financial shocks combined can explain about two-thirds (9 percentage points out of a 13 percent decline) of the output decline from 2007 to the end of 2010.

In order to judge the success of the model, at least from a statistical standpoint, I run a formal comparison between the estimated model and an estimated version of the model without banks. To this end, I estimate (using the same priors and data) a version of the model without banks, and perform a standard Bayesian model comparison between the two models. In the model without banks, there is no capital adequacy constraint, savings can be transformed into loans at no cost, and financial intermediation is performed by household savers directly. As a consequence, quadratic adjustment costs aside, interest rate spreads equal zero at all times. Under the assumption that both models are viewed as equally likely a

**Table 3**  
Historical decomposition.

	2007	2008	2009	2010	2007–2010
<b>Contribution to Output</b>					
Default shocks	-0.2	-1.2	-1.4	0.1	-2.7
Housing Demand shock	-1.3	-1.7	-1.0	0.0	-4.1
LTV shocks	1.1	0.2	-2.2	-1.5	-2.4
Preference shock	2.9	-0.1	-4.9	2.6	0.5
TFP shocks	-2.2	-0.8	0.3	-1.3	-4.0
<b>All shocks (data)</b>	<b>0.3</b>	<b>-3.6</b>	<b>-9.3</b>	<b>-0.1</b>	<b>-12.6</b>
<b>Contribution to Investment</b>					
Default shocks	-0.5	-2.7	-3.0	0.7	-5.5
Housing Demand shock	-2.1	-3.4	-2.8	-0.9	-9.1
LTV shocks	3.5	1.7	-6.8	-5.7	-7.3
Preference shock	2.5	-0.9	-5.7	5.1	1.0
TFP shocks	-0.5	1.1	-4.9	2.2	-2.1
<b>All shocks (data)</b>	<b>3.0</b>	<b>-4.2</b>	<b>-23.3</b>	<b>1.4</b>	<b>-23.1</b>
<b>Contribution to Consumption</b>					
Default shocks	-0.1	-0.7	-0.9	-0.1	-1.7
Housing Demand shock	-1.1	-1.1	-0.4	0.3	-2.3
LTV shocks	0.2	-0.3	-0.6	-0.1	-0.7
Preference shock	3.1	0.1	-4.6	1.8	0.4
TFP shocks	-2.7	-1.4	2.0	-2.5	-4.7
<b>All shocks (data)</b>	<b>-0.6</b>	<b>-3.4</b>	<b>-4.5</b>	<b>-0.6</b>	<b>-9.1</b>

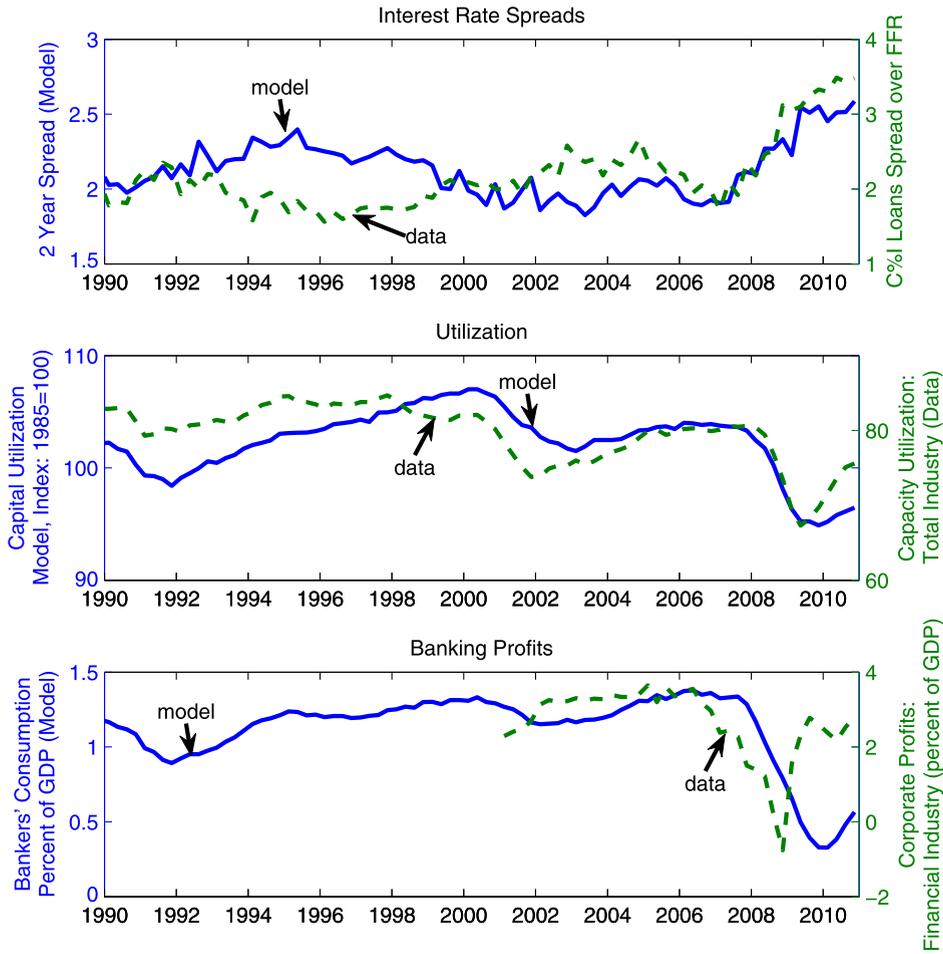
Note: Contribution of each estimated shock to year-on-year growth in Annual Output (sum of consumption and nonresidential fixed investment), Annual Investment and Annual Consumption.



**Fig. 3.** Historical decomposition of the estimated model. Note: The solid lines plot actual data. The bars show the contributions of the estimated financial shocks. Data are expressed in deviation from their mean.

priori, I obtain a posterior odds ratio of about  $e^{4.5}$  that strongly favors (in the sense Kass and Raftery, 1995) the model with banks.

As an additional test of the empirical fit of the model, I conduct an external validation exercise to assess the reliability of the model in fitting time series that were not used as inputs in the estimation. Such an exercise is of particular interest since it addresses the critique that DSGE models can do a good job at fitting the data in sample, but have poor performance



**Fig. 4.** External validation: historical decomposition of model series. Note: The solid lines plot model simulated (smoothed estimates) series. The dashed lines plot similar objects from actual data.

otherwise. In particular, given the estimated shocks, I contrast in Fig. 4 the model's simulated time series for interest rate spreads, capacity utilization and bankers' consumption against their data counterparts. The top panel plots the two-year ahead interest rate spread against the C&I Loan Rate Spread for all loans from the Fed Survey of Terms of Business Lending.<sup>12</sup> Both in the model and in the data, the interest-rate spread rises markedly during the 2007–2009 period, although the increase – in percentage terms – is slightly larger in the data than in the model.<sup>13</sup> In the middle panel, the behavior of capital utilization in the model mimics its data analogue,<sup>14</sup> with both the model and the data pointing to a large and persistent decline in utilization around the financial crisis. The bottom panel compares bankers' consumption with a measure of the health of the banking system in the data, namely corporate profits of the financial sector.<sup>15</sup> Both measures tank during the Great Recession.

#### 4.2. The transmission mechanism of financial shocks

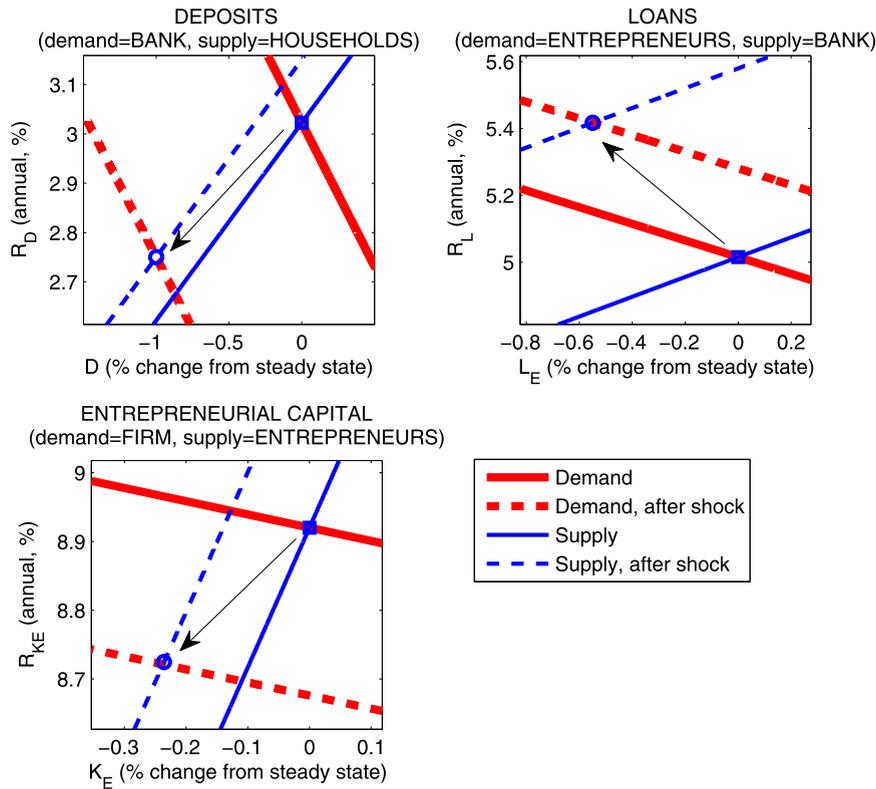
Fig. 5 illustrates the model's transmission mechanism for three key markets, at the model's parameter estimates, by plotting the model-consistent demand and supply curves derived from the relevant Euler equations. I focus on how resources

<sup>12</sup> The series name in the data is *FCIRS@USECON*. I construct the model interest spread as the difference between the lending rate for entrepreneurs ( $R_E$ ) and the deposit rate ( $R_H$ ). I construct a model-consistent two-year spread using the expectations hypothesis to match the average duration of C&I Loans in the Survey of Terms of Business Lending.

<sup>13</sup> In the model, spreads rise when banks' financial conditions worsen, since they signal the unwillingness of banks to lend funds. In the data, the rise in spreads reflects default risk that is not priced in the model.

<sup>14</sup> There is no satisfactory counterpart to the model's capital utilization in the data. Existing data refer only to manufacturing, and are calculated by comparing actual production with a measure of full-capacity production. The proxy I use is the total industry capacity utilization is the Board of Governors of the Federal Reserve System (Industrial Production and Capacity Utilization Summary Table, *CUT@USECON*).

<sup>15</sup> The data source for corporate profits is the BEA GDP release. The series name is *YCPDF@USECON*.



**Fig. 5.** Transmission mechanism of default shock. Note: Each panel plots the linearized demand and supply curves for deposits, loans to entrepreneurs, and entrepreneurial capital in steady state and away from it, in the first period when a redistribution shock (that transfers 2 percent of GDP from banks to impatient households) hits.

are transferred from the savers (the patient households) to the ultimate users of them (the final good firms), and on how a given size financial shock affect the functioning of these markets. I focus on a redistribution shock that leads to a rise in charge-off rates for household loans from 0 to 2 percent, a magnitude in line with the Great Recession. In the market for deposits  $D_t$ , household-savers set aside resources, and supply them to the bank. The bank demands deposits from the household. The slopes of demand and supply curves are a function of the estimated parameters  $\phi_{DB}$  and  $\phi_{DH}$ , which measure the convex adjustment cost of changing deposits for banks and households. The linearized demand and supply schedules are plotted in the figure. The negative financial shock hits the financial position of the bank and – holding everything else the same – reduces the bank’s ability to borrow from the household at a given deposit rate. The deposits demand curve shifts to the left, thus reducing equilibrium deposits and the deposit interest rate.<sup>16</sup>

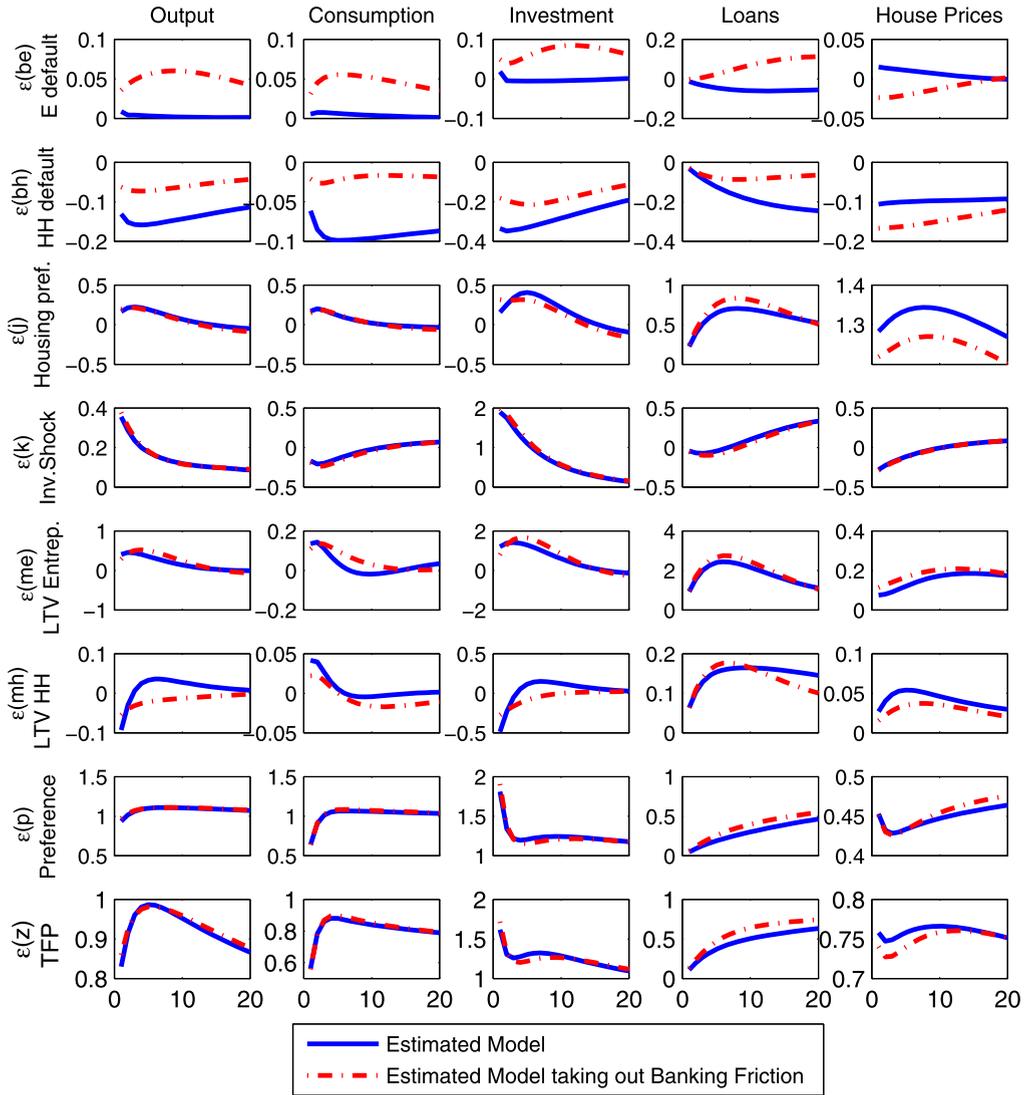
In the market for loans  $L_E$ , the dynamics reflect two forces. On the supply side, as bankers are forced to deleverage, they reduce the supply of loans, which shifts inwards. On the demand side, at the going interest rate, entrepreneurs would like to borrow more: given their high discount factor and their binding borrowing constraint, the drop in consumption growth increases their loan demand. At the model’s estimates, the inward shift in loan supply is far larger than the increase in loan demand, the equilibrium lending rate rises, and total loans decline.

In the market for capital  $K_E$ , as equilibrium borrowing drops, entrepreneurs are less able to supply funds to final good firms, and the supply of capital drops. Capital demand also drops because wealthier borrowers decide to work less, and because factor complementarities reduce the marginal product of capital as real estate demand and utilization rates fall, even as total factor productivity remains unchanged. In turn, the decline in the demand for other factors lowers the marginal product of capital, thus further exacerbating the decline of output.

#### 4.3. Impulse response analysis

Fig. 6 offers a summary picture of the model dynamics in response to the estimated shocks, at the mean of estimated parameter values. To better highlight the role of banks, I compare the model responses to those of a model without banks that retains financial frictions on households and firms.

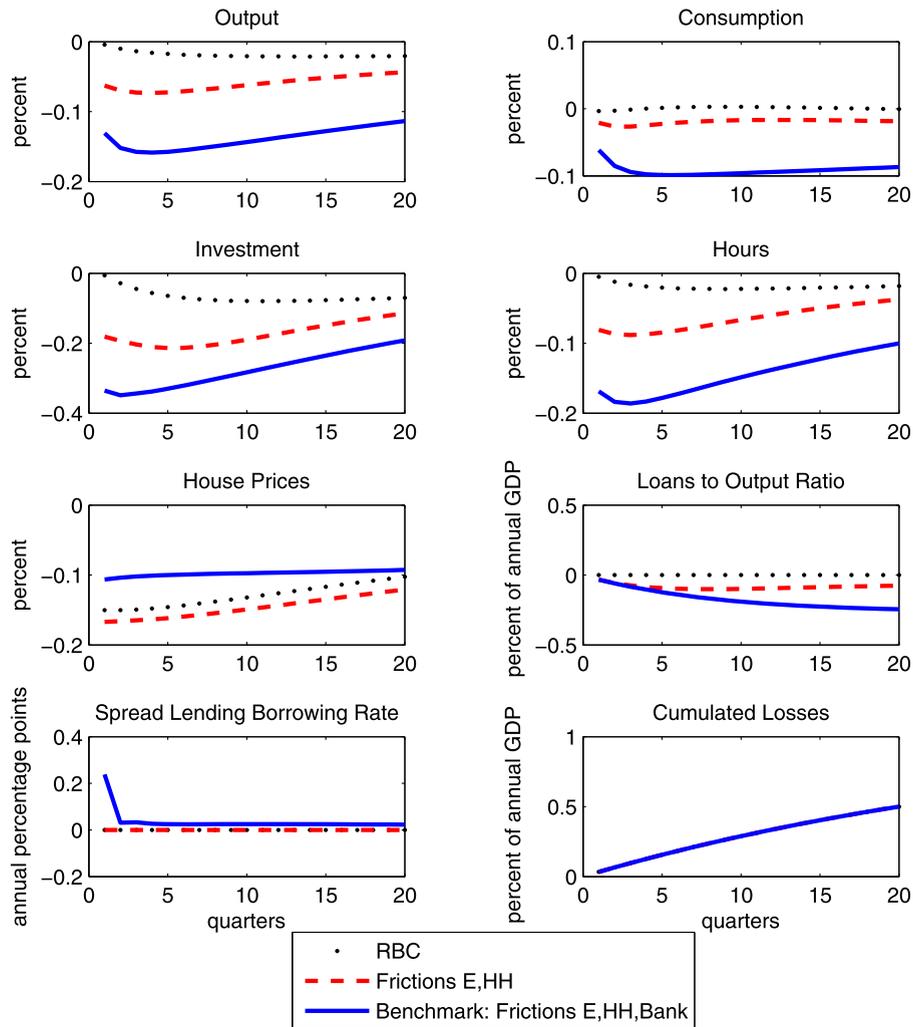
<sup>16</sup> As general equilibrium repercussions affect wages and consumption, the household’s supply of deposits – which depends on interest rates and expected consumption growth – moves too. In particular, depending on the persistence of the shock and the habit coefficient, the supply of deposits may either increase or decrease. At the model’s estimates, the supply of deposits is reduced, thus partly mitigating the decline in deposit rates.



**Fig. 6.** Impulse responses to all shocks, estimated banking model and counterfactual model without banks. Note: horizontal axis: quarters from the shock; vertical axis: percent deviation from the steady state. The solid lines plot, for each row, the responses to each estimated shock, one standard deviation in size. The dashed lines plot the same model shutting off the banking sector.

The top two rows show the impulse response to repayment shocks of entrepreneurs and impatient households respectively, and illustrate how the presence of leveraged banks amplifies such shocks. In particular, the second row shows how a one standard deviation household repayment shock (corresponding to a persistent rise in charge-off rates for the banks of 0.13 percentage points) leads to a protracted decline in output and investment, whereas the effects would be more muted in a frictionless model without banks. In other words, the presence of constrained banks produces larger negative effects on output for given redistribution shocks that transfer resources away from banks. These effects are present both when the redistribution works in favor of entrepreneurs – first row – and when it works in favor of household borrowers – second row –, but they are weaker in the first case. For when resources are transferred to entrepreneurs, the reduction in loan supply stemming from the reduction in bankers’ net worth is partly offset by the increase in investment and capital accumulation due to higher entrepreneurial net worth, thus mitigating the output decline.

As for the responses to other shocks, the dynamics in the model with banks are not drastically different from those of the model without banks. This implies that financial frictions on banks work mostly to amplify shocks affecting banks’ net worth, but matter relatively less for traditional business-cycle shocks. To understand this result, it is useful to consider that capital-constrained banks create a static and dynamic wedge that limits the amount of savings that can be transformed into investment goods. When given shocks move this wedge by little, the dynamics of the two models are similar. However, the redistribution shocks directly affect the wedge through their strong effect on bank net worth. There are some additional, subtle differences between the two models that do not show up in the dynamics, but are important for the steady-state



**Fig. 7.** Impulse responses to estimated household default shock. Note: horizontal axis: quarters from the shock; vertical axis: percentage deviation from the steady state. Loans and loan losses are percentages of annualized output. The Spread between the Loan and Deposit Rate is expressed in annualized percentage points. The shock is one standard deviation in size.

implications of the two models. The capital requirement on banks constrains the amount of savings that can be transformed into investment goods. This constraint is absent in the model without banks, which implicitly assumes that all savings can be transformed into investment goods at no cost (except for the standard quadratic adjustment costs). For this reason, at the estimated parameters' mode, steady-state consumption, investment and output are, respectively, 0.5, 4.5 and 1.5 percent higher in the model without banks than in the model with banks.

Fig. 7 illustrates the strength of the various channels in shaping output dynamics in response to an estimated one standard deviation household default shock. I compare three models: the RBC model; a model with traditional financial frictions on both firms and households; and my model, which combines financial and banking frictions.

The RBC model has only two household types, all investment is done by the patient households, and the entrepreneurial sector is shut off (by setting  $\mu$  and  $\nu$  to zero). The only friction pertains to the fact that households who borrow are financially constrained: if this friction was missing, there would be no heterogeneity, and no way to think about redistribution shocks (the shock would wash out in the aggregate, in an accounting and behavioral sense). In the RBC version, the redistribution shock transfers wealth from the savers to the borrowers. Accordingly, borrowers consume more. Patient households, instead, consume less, but reduce their saving in order to smooth their consumption. All told, the decline in savers' consumption does not fully offset the rise in borrowers' consumption, and aggregate consumption rises. In turn, lower savings lead to a decline in investment that more than offsets the rise in consumption, so that aggregate output falls, although the total effects are very small. A one standard deviation shock leads to a 0.02 percent decline in output after one year.

In the model with financial frictions both on households and on entrepreneurs, but without banks, the decline in households' saving following the repayment shock reduces the supply of available funds for the entrepreneurs, and causes a

knock-on effect on borrowing and investment that further magnifies the output decline. The decline in output after one year is about 0.05 percent, twice as large than in the RBC case.

The largest negative effects on economic activity from the repayment shock occur when both the banking channel and the collateral channel are at work, thus restoring the benchmark model with leveraged banks. By putting direct pressure on the bank's balance sheet, the repayment shock further strengthens the drop in output. At the trough, the output decline is 0.15 percent, almost one order of magnitude larger than in the model without financial frictions.

## 5. Concluding remarks

In this paper I have presented and estimated a DSGE model where losses sustained by banks can produce sizeable, pronounced and long-lasting effects on business activity. The key ingredients of the model are constraints on the leverage of the banks and a business sector that is bank-dependent for its operations. In an estimated version of the model, financial shocks account for about two-thirds of the decline in output during the Great Recession.

Despite its complexity, my model precludes an examination of certain aspects that may be important to understand the role of banks and leveraged agents in business fluctuations. First, banks offer the important benefit of maturity transformation by intermediating across needs and projects with different termination dates. However, while the simple model of this paper features loans and deposits with different adjustment costs, it abstract from a richer examination of the liquidity role provided of banks through this function. Second, because of the illiquid nature of many of the bank's assets, banks can be subject to runs, especially in periods when their balance sheets are weak or perceived as such. Third, default episodes are obviously the consequence of some negative shocks hitting elsewhere in the economy, and one would love to have a parsimonious macro framework that explains defaults without losing the tractability of a stylized model that can be taken to the data. The recent papers by [Andreasen et al. \(2013\)](#), [Forlatti and Lambertini \(2011\)](#) and [Gertler and Kiyotaki \(2013\)](#) contain interesting examples of models that have begun to address these issues.

## Appendix A. Complete set of equations of the basic model

The basic model is described by the following set of equations. I denote with  $u_{ij}$  the marginal utility of good  $i$  for agent  $j$ .

$$C_{H,t} + D_t + q_t(H_{H,t} - H_{H,t-1}) = R_{H,t-1}D_{t-1} + W_{H,t}N_{H,t} + \varepsilon_{H,t}, \quad (\text{A.1})$$

$$u_{CH,t} = \beta_H E_t(R_{H,t}u_{CH,t+1}), \quad (\text{A.2})$$

$$W_{H,t}u_{CH,t} = \frac{\tau}{1 - N_{H,t}}, \quad (\text{A.3})$$

$$q_t u_{CH,t} = u_{HH,t} + \beta_H E_t(q_{t+1}u_{CH,t+1}), \quad (\text{A.4})$$

$$C_{B,t} + R_{H,t-1}D_{t-1} + L_{E,t} + ac_{EB,t} = D_t + R_{E,t}L_{E,t-1} - \varepsilon_{H,t}, \quad (\text{A.5})$$

$$D_t = \gamma(L_{E,t} - E_t\varepsilon_{H,t+1}), \quad (\text{A.6})$$

$$\left(1 - \gamma + \frac{\partial ac_{EB,t}}{\partial L_{E,t}}\right)u_{CB,t} = \beta_B E_t((R_{E,t+1} - \gamma R_{H,t})u_{CB,t+1}), \quad (\text{A.7})$$

$$C_{E,t} + q_t(H_{E,t} - H_{E,t-1}) + R_{E,t}L_{E,t-1} + W_{H,t}N_{H,t} = Y_t + L_{E,t} + ac_{EE,t}, \quad (\text{A.8})$$

$$Y_t = H_{E,t-1}^v N_{H,t}^{1-v}, \quad (\text{A.9})$$

$$L_{E,t} = m_H E_t \left( \frac{q_{t+1}}{R_{E,t+1}} H_{E,t} \right) - m_N W_{H,t} N_{H,t}, \quad (\text{A.10})$$

$$\left( q_t - E_t \left( \left( 1 - \frac{\partial ac_{EE,t}}{\partial L_{E,t}} \right) \frac{m_H q_{t+1}}{R_{E,t+1}} \right) \right) u_{CE,t} = \beta_E E_t \left( \left( q_{t+1} (1 - m_H) + v \frac{Y_{t+1}}{H_{E,t}} \right) u_{CE,t+1} \right), \quad (\text{A.11})$$

$$(1 - v)Y_t = W_{H,t}N_{H,t}E_t \left( 1 + m_N \left( 1 - \frac{\partial ac_{EE,t}}{\partial L_{E,t}} - \beta_E R_{E,t+1} \frac{u_{CE,t+1}}{u_{CE,t}} \right) \right), \quad (\text{A.12})$$

$$H_{H,t} + H_{E,t} = 1. \quad (\text{A.13})$$

The model endogenous variables are  $Y$ ,  $H_E$ ,  $H_H$ ,  $N_H$ ,  $C_B$ ,  $C_E$ ,  $C_H$ ,  $L_E$ ,  $D$ ,  $q$ ,  $W_H$ ,  $R_E$ , and  $R_H$ . The exogenous repayment shock is  $\varepsilon_{H,t}$ .

## Appendix B. Complete set of equations of the extended model

This section describes in detail the extended model. Unless stated otherwise, the absence of subscript from a variable denotes the steady state of that variable. For instance,  $K_{H,t}$  is household capital at time  $t$ , and  $K_H$  is the steady-state value of household capital.

### B.1. Patient households

Patient households solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta_H^t (A_{p,t} (1 - \eta) \log(C_{H,t} - \eta C_{H,t-1}) + j A_{j,t} A_{p,t} \log H_{H,t} + \tau \log(1 - N_{H,t}))$$

subject to:

$$\begin{aligned} C_{H,t} + \frac{K_{H,t}}{A_{K,t}} + D_t + q_t(H_{H,t} - H_{H,t-1}) + ac_{KH,t} + ac_{DH,t} \\ = \left( R_{M,t} z_{KH,t} + \frac{1 - \delta_{KH,t}}{A_{K,t}} \right) K_{H,t-1} + R_{H,t-1} D_{t-1} + W_{H,t} N_{H,t}, \end{aligned} \quad (B.1)$$

where the adjustment costs take the following form

$$\begin{aligned} ac_{KH,t} &= \frac{\phi_{KH}}{2} \frac{(K_{H,t} - K_{H,t-1})^2}{K_H}, \\ ac_{DH,t} &= \frac{\phi_{DH}}{2} \frac{(D_t - D_{t-1})^2}{D}, \end{aligned}$$

and the depreciation function is

$$\delta_{KH,t} = \delta_{KH} + b_{KH} (0.5 \zeta'_H z_{KH,t}^2 + (1 - \zeta'_H) z_{KH,t} + (0.5 \zeta'_H - 1)),$$

where  $\zeta'_H = \frac{\zeta_H}{1 - \zeta_H}$  is a parameter measuring the curvature of the utilization rate function.  $\zeta_H = 0$  implies  $\zeta'_H = 0$ ;  $\zeta_H$  approaching 1 implies  $\zeta'_H$  approaches infinity and  $\delta_{KH,t}$  stays constant.  $b_{KH} = \frac{1}{\beta_H} + 1 - \delta_{KH}$  and implies a unitary steady-state utilization rate.  $ac_t$  measures a quadratic adjustment cost for changing the quantity  $i$  between time  $t - 1$  and time  $t$ . Both habits and adjustment costs are assumed to be external.

Denote with  $u_{CH,t} = \frac{A_{p,t}(1-\eta)}{C_{H,t}-\eta C_{H,t-1}}$  and  $u_{HH,t} = \frac{j A_{j,t} A_{p,t}}{H_{H,t}}$  the marginal utilities of consumption and housing. The optimality conditions yield equations for deposit supply, labor supply, supply of capital, housing demand, and for the optimal utilization rate:

$$u_{CH,t} \left( 1 + \frac{\partial ac_{DH,t}}{\partial D_t} \right) = \beta_H E_t (R_{H,t} u_{CH,t+1}), \quad (B.2)$$

$$W_{H,t} u_{CH,t} = \frac{\tau}{1 - N_{H,t}}, \quad (B.3)$$

$$\frac{1}{A_{K,t}} u_{CH,t} \left( 1 + \frac{\partial ac_{KH,t}}{\partial K_{H,t}} \right) = \beta_H E_t \left( \left( R_{M,t+1} z_{KH,t+1} + \frac{1 - \delta_{KH,t+1}}{A_{K,t+1}} \right) u_{CH,t+1} \right), \quad (B.4)$$

$$q_t u_{CH,t} = u_{HH,t} + \beta_H E_t (q_{t+1} u_{CH,t+1}), \quad (B.5)$$

$$R_{M,t} = \frac{\partial \delta_{KH,t}}{\partial z_{KH,t}}, \quad (B.6)$$

where  $A_{K,t}$  is an investment shock,  $A_{p,t}$  is a consumption preference shock,  $A_{j,t}$  is a housing demand shock.

### B.2. Impatient households

Impatient households solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta_S^t (A_{p,t} (1 - \eta) \log(C_{S,t} - \eta C_{S,t-1}) + j A_{j,t} A_{p,t} \log H_{S,t} + \tau \log(1 - N_{S,t})),$$

where

$$\beta_S < \left( 1 - ((1 - \beta_B) \rho_D + (1 - \rho_D) \gamma_S) \frac{1 - \beta_B R_H}{1 - \beta_B \rho_D} \right) \beta_B,$$

subject to

$$C_{S,t} + q_t (H_{S,t} - H_{S,t-1}) + R_{S,t-1} L_{S,t-1} - \varepsilon_{H,t} + ac_{SS,t} = L_{S,t} + W_{S,t} N_{S,t}, \quad (B.7)$$

and to

$$L_{S,t} \leq \rho_S L_{S,t-1} + (1 - \rho_S) m_S A_{MH,t} E_t \left( \frac{q_{t+1}}{R_{S,t}} H_{S,t} \right), \quad (\text{B.8})$$

where  $\varepsilon_{H,t}$  is the borrower repayment shock,  $A_{MH,t}$  is a loan-to-value ratio shock. The adjustment cost is:

$$ac_{SS,t} = \frac{\phi_{SS}}{2} \frac{(L_{S,t} - L_{S,t-1})^2}{L_S}.$$

The first-order conditions are, denoting with  $u_{CS,t} = \frac{A_{p,t}(1-\eta)}{C_{S,t}-\eta C_{S,t-1}}$  and  $u_{HS,t} = \frac{j A_{j,t} A_{p,t}}{H_{S,t}}$  the marginal utilities of consumption and housing, and with  $\lambda_{S,t}$  the multiplier on the borrowing constraint normalized by the marginal utility of consumption:

$$\left( 1 - \frac{\partial ac_{SS,t}}{\partial L_{S,t}} - \lambda_{S,t} \right) u_{CS,t} = \beta_S E_t \left( (R_{S,t} - \rho_S \lambda_{S,t+1}) u_{CS,t+1} \right), \quad (\text{B.9})$$

$$W_{S,t} u_{CS,t} = \frac{\tau_S}{1 - N_{S,t}}, \quad (\text{B.10})$$

$$\left( q_t - \lambda_{S,t} (1 - \rho_S) m_S A_{MH,t} E_t \frac{q_{t+1}}{R_{S,t}} \right) u_{CS,t} = u_{HS,t} + \beta_S E_t (q_{t+1} u_{CS,t+1}). \quad (\text{B.11})$$

### B.3. Bankers

Bankers solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta_B^t (1 - \eta) \log(C_{B,t} - \eta C_{B,t-1})$$

where

$$\beta_B < \beta_H,$$

subject to

$$C_{B,t} + R_{H,t-1} D_{t-1} + L_{E,t} + L_{S,t} + ac_{DB,t} + ac_{EB,t} + ac_{SB,t} = D_t + R_{E,t} L_{E,t-1} + R_{S,t} L_{S,t-1} - \varepsilon_{E,t} - \varepsilon_{H,t}, \quad (\text{B.12})$$

where  $\varepsilon_{E,t}$  is the entrepreneur repayment shock. The adjustment costs are:

$$ac_{DB,t} = \frac{\phi_{DB}}{2} \frac{(D_t - D_{t-1})^2}{D},$$

$$ac_{EB,t} = \frac{\phi_{EB}}{2} \frac{(L_{E,t} - L_{E,t-1})^2}{L_E},$$

$$ac_{SB,t} = \frac{\phi_{SB}}{2} \frac{(L_{S,t} - L_{S,t-1})^2}{L_S}.$$

Denote  $\varepsilon_t = \varepsilon_{E,t} + \varepsilon_{H,t}$ . Let  $L_t = L_{E,t} + L_{S,t}$ . The banker's constraint is a capital adequacy constraint of the form:

$$(L_t - D_t - E_t \varepsilon_{t-1})_{\text{bank equity}} \geq \rho_D (L_{t-1} - D_{t-1} - E_{t-1} \varepsilon_t) + (1 - \gamma)(1 - \rho_D)(L_t - E_t \varepsilon_{t+1})_{\text{bank assets}},$$

stating that bank equity (after expected losses) must exceed a fraction of bank assets, allowing for a partial adjustment in bank capital given by  $\rho_D$ . Such constraint can be rewritten as a leverage constraint of the form

$$D_t \leq \rho_D (D_{t-1} - (L_{E,t-1} + L_{S,t-1} - E_{t-1}(\varepsilon_{E,t} + \varepsilon_{H,t}))) + (1 - (1 - \gamma)(1 - \rho_D))(L_{E,t} + L_{S,t} - E_t(\varepsilon_{E,t+1} + \varepsilon_{H,t+1})). \quad (\text{B.13})$$

The first order conditions to the banker's problem imply, choosing  $D$ ,  $L_E$ ,  $L_S$  and denoting with  $\lambda_{B,t}$  be the multiplier on the borrowing constraint normalized by  $u_{CB,t}$ , the banker's marginal utility of consumption:

$$\left( 1 - \lambda_{B,t} - \frac{\partial ac_{DB,t}}{\partial D_t} \right) u_{CB,t} = \beta_B E_t \left( (R_{H,t} - \rho_D \lambda_{B,t+1}) u_{CB,t+1} \right), \quad (\text{B.14})$$

$$\left( 1 - (\gamma_E(1 - \rho_D) + \rho_D) \lambda_{B,t} + \frac{\partial ac_{EB,t}}{\partial L_{E,t}} \right) u_{CB,t} = \beta_B E_t \left( (R_{E,t+1} - \rho_D \lambda_{B,t+1}) u_{CB,t+1} \right), \quad (\text{B.15})$$

$$\left( 1 - (\gamma_S(1 - \rho_D) + \rho_D) \lambda_{B,t} + \frac{\partial ac_{SB,t}}{\partial L_{S,t}} \right) u_{CB,t} = \beta_B E_t \left( (R_{S,t} - \rho_D \lambda_{B,t+1}) u_{CB,t+1} \right). \quad (\text{B.16})$$

#### B.4. Entrepreneurs

Entrepreneurs obtain loans and produce goods (including capital). Entrepreneurs hire workers and demand capital supplied by the household sector. They maximize

$$\max E_0 \sum_{t=0}^{\infty} \beta_E^t (1 - \eta) \log(C_{E,t} - \eta C_{E,t-1})$$

where

$$\beta_E \left( 1 - ((1 - \beta_B)\rho_D + (1 - \rho_D)\gamma_E) \frac{1 - \beta_B R_H}{1 - \beta_B \rho_D} \right) < \beta_B,$$

subject to:

$$\begin{aligned} C_{E,t} + \frac{K_{E,t}}{A_{K,t}} + q_t H_{E,t} + R_{E,t} L_{E,t-1} + W_{H,t} N_{H,t} + W_{S,t} N_{S,t} + R_{M,t} z_{KH,t} K_{H,t-1} + ac_{KE,t} + ac_{EE,t} \\ = Y_t + \frac{1 - \delta_{KE,t}}{A_{K,t}} K_{E,t-1} + q_t H_{E,t-1} + L_{E,t} + \varepsilon_{E,t}, \end{aligned} \quad (B.17)$$

and to

$$Y_t = A_{Z,t} (z_{KH,t} K_{H,t-1})^{\alpha(1-\mu)} (z_{KE,t} K_{E,t-1})^{\alpha\mu} H_{E,t-1}^v N_{H,t}^{(1-\alpha-\nu)(1-\sigma)} N_{S,t}^{(1-\alpha-\nu)\sigma}, \quad (B.18)$$

where  $A_{Z,t}$  is a shock to total factor productivity. The adjustment costs are

$$\begin{aligned} ac_{KE,t} &= \frac{\phi_{KE}}{2} \frac{(K_{E,t} - K_{E,t-1})^2}{K_E}, \\ ac_{EE,t} &= \frac{\phi_{EE}}{2} \frac{(L_{E,t} - L_{E,t-1})^2}{L_E}. \end{aligned}$$

Note that symmetrically to the household problem entrepreneurs are subject to an investment shock, can adjust the capital utilization rate, and pay a quadratic capital adjustment cost. The depreciation rate is governed by

$$\delta_{KE,t} = \delta_{KE} + b_{KE} (0.5 \zeta_E' z_{KE,t}^2 + (1 - \zeta_E') z_{KE,t} + (0.5 \zeta_E' - 1)),$$

where setting  $b_{KE} = \frac{1}{\beta_E} (1 - \lambda_E (1 - \rho_E) m_K) - (1 - \delta_{KE})$  implies a unitary steady state utilization rate.

Entrepreneurs are subject to a borrowing/pay in advance constraint that acts as a wedge on the capital and labor demand. The constraint is

$$L_{E,t} = \rho_E L_{E,t-1} + (1 - \rho_E) A_{ME,t} E_t \left( m_H \frac{q_{t+1}}{R_{E,t+1}} H_{E,t} + m_K K_{E,t} - m_N (W_{H,t} N_{H,t} + W_{S,t} N_{S,t}) \right). \quad (B.19)$$

Letting  $u_{CE,t}$  be the marginal utility of consumption and  $\lambda_{E,t}$  the borrowing constraint normalized by the marginal utility of consumption  $u_{CE,t}$ , the first order conditions for loans, capital and real estate are:

$$\left( 1 - \lambda_{E,t} - \frac{\partial ac_{EE,t}}{\partial L_{E,t}} \right) u_{CE,t} = \beta_E E_t ((R_{E,t+1} - \rho_E \lambda_{E,t+1}) u_{CE,t+1}), \quad (B.20)$$

$$\left( 1 + \frac{\partial ac_{KE,t}}{\partial K_{E,t}} - \lambda_{E,t} (1 - \rho_E) m_K A_{ME,t} \right) u_{CE,t} = \beta_E E_t ((1 - \delta_{KE,t+1} + R_{K,t+1} z_{KE,t+1}) u_{CE,t+1}), \quad (B.21)$$

$$\left( q_t - \lambda_{E,t} (1 - \rho_E) m_H A_{ME,t} E_t \left( \frac{q_{t+1}}{R_{E,t+1}} \right) \right) u_{CE,t} = \beta_E E_t (q_{t+1} (1 + R_{V,t+1}) u_{CE,t+1}). \quad (B.22)$$

Additionally, these conditions can be combined with those of the production arm of the firm, giving:

$$\alpha \mu Y_t = R_{K,t} z_{KE,t} K_{E,t-1}, \quad (B.23)$$

$$\alpha (1 - \mu) Y_t = R_{M,t} z_{KH,t} K_{H,t-1}, \quad (B.24)$$

$$\nu Y_t = R_{V,t} q_t H_{E,t-1}, \quad (B.25)$$

$$(1 - \alpha - \nu)(1 - \sigma) Y_t = W_{H,t} N_{H,t} (1 + (1 - \rho_E) m_N A_{ME,t} \lambda_{E,t}), \quad (B.26)$$

$$(1 - \alpha - \nu) \sigma Y_t = W_{S,t} N_{S,t} (1 + (1 - \rho_E) m_N A_{ME,t} \lambda_{E,t}), \quad (B.27)$$

$$R_{K,t} = \frac{\partial \delta_{KE,t}}{\partial z_{KE,t}}. \quad (B.28)$$

### B.5. Equilibrium

Market clearing is implied by Walras's law by aggregating all the budget constraints. For housing, we have the following market clearing condition

$$H_{H,t} + H_{S,t} + H_{E,t} = 1. \quad (\text{B.29})$$

The model endogenous variables are  $Y, H_E, H_H, H_S, K_E, K_H, N_H, N_S, C_B, C_E, C_H, z_{KH}, z_{KE}, L_E, L_S, D, q, W_H, W_S, R_K, R_M, R_V, R_E, R_S, R_H, \lambda_E, \lambda_S, \lambda_B$ , together with the definition of the depreciation rate functions and the adjustment cost functions given in the text above.

### B.6. Shocks

The following zero-mean, AR(1) shocks are present in the estimated version of the model:  $\varepsilon_E, \varepsilon_H, A_j, A_K, A_{ME}, A_{MH}, A_p, A_z$ . The shocks follow the processes given by:

$$\begin{aligned} \varepsilon_{E,t} &= \rho_{be} \varepsilon_{E,t-1} + \nu_{E,t}, & \nu_E &\sim N(0, \sigma_{be}), \\ \varepsilon_{H,t} &= \rho_{bh} \varepsilon_{H,t-1} + \nu_{H,t}, & \nu_H &\sim N(0, \sigma_{bh}), \\ \log A_{j,t} &= \rho_j \log A_{j,t-1} + \nu_{j,t}, & \nu_j &\sim N(0, \sigma_j), \\ \log A_{K,t} &= \rho_K \log A_{K,t-1} + \nu_{K,t}, & \nu_K &\sim N(0, \sigma_K), \\ \log A_{ME,t} &= \rho_{me} \log A_{ME,t-1} + \nu_{ME,t}, & \nu_{ME} &\sim N(0, \sigma_{me}), \\ \log A_{MH,t} &= \rho_{mh} \log A_{MH,t-1} + \nu_{MH,t}, & \nu_{MH} &\sim N(0, \sigma_{mh}), \\ \log A_{p,t} &= \rho_p \log A_{p,t-1} + \nu_{p,t}, & \nu_p &\sim N(0, \sigma_p), \\ \log A_{z,t} &= \rho_z \log A_{z,t-1} + \nu_{z,t}, & \nu_z &\sim N(0, \sigma_z). \end{aligned}$$

## Appendix C. Estimation: data construction

The model is estimated with U.S. quarterly data.

I use the following time series as observables. Series mnemonics are from Haver Analytics. Consumption and Investment data are from NIPA. Loan data are from the Flow of Funds Accounts. Loan charge-offs data are from the Federal Reserve Board.

### 1. Consumption

Model variable:  $C_t$ .

Data: CH@USECON: Real Personal Consumption Expenditures (SAAR, Bil.Chn.2005\$, Source: BEA). The series is log transformed, and detrended with a quadratic trend.

### 2. Investment

Model variable:  $I_t = \frac{K_{E,t} - (1 - \delta_{KE,t})K_{E,t-1} + K_{H,t} - (1 - \delta_{KH,t})K_{H,t-1}}{A_{K,t}}$ .

Data: FNH@USECON: Real Private Nonresidential Fixed Investment (SAAR, Bil.Chn.2005\$, Source: BEA). The series is log transformed, and detrended with a quadratic trend.

### 3. Losses on loans to entrepreneurs

Model variable:  $\varepsilon_{E,t}$ .

Data:  $\varepsilon_{E,t} = \text{DYRM} \times \text{OL14MOR5} + \text{DYI} \times (\text{OL14OTL5} + \text{OL14BLN5})$ ,

where: DYRM@USECON: Loan Charge-Off Rate: Commercial Real Estate Loans: All Comm'l Banks (SAAR, %) (Source: H8 Release, Federal Reserve Board);

OL14BLN5@FFUNDS: Nonfinancial business; total mortgages; liability (Source: Table L.101, Flow of Funds Accounts);

DYI@USECON: Loan Charge-Off Rate: C&I Loans: All Insured Comm'l Banks (SAAR, %) (Source: H8 Release, Federal Reserve Board);

OL14OTL5@FFUNDS: Nonfinancial business; other loans and advances; liability (Source: Table L.101, Flow of Funds Accounts);

OL14BLN5@FFUNDS: Nonfinancial business; depository institution loans n.e.c.; liability (Source: Table L.101, Flow of Funds Accounts).

The data series is constructed multiplying commercial bank charge-off rates by the volume of loans (C&I loans, mortgages and loans not elsewhere classified) held by nonfinancial businesses.

Both in the model and in the data, charge-offs rates are scaled by steady-state GDP. In the data, liabilities are in dollars and steady-state GDP is measured by a cubic trend in the sum of nominal consumption and investment.

Notes: When a bank loan is securitized and sold to another bank or GSE, it shows as a loan in the liability side of the nonfinancial business sector balance sheet, while it shows as a security in the asset side of the bank balance sheet.

Charge-offs are measured in the data by looking at reported losses of banks on loans on the asset side of the balance sheet. By multiplying charge-off rates by the total amount of liabilities of the business sector in the form of loans, one is implicitly allocating losses to all loans and securities held by banks or institutions who purchased securities whose underlying asset are these loans (alternatively, one is consolidating GSE, commercial banks and ABS issuers into one single, big, financial institution). More detail is provided in [Appendix D](#).

Charge-offs for commercial mortgages (*DYRM*) are available starting in 1991Q1, whereas charge-offs for C&I Loans (*DYI*) begin in 1985Q1. I use the regression coefficients of a regression of *DYRM* on a constant and *DYI* for the 1991–2010 period and data on *DYI* in order to backcast the missing data for *DYRM* for the 1986–1990 period.

#### 4. Losses on loans to households

*Model variable:*  $\varepsilon_{H,t}$ .

*Data:*  $\varepsilon_{Ht} = DYRR \times XL15HOM5 + DYU \times HCCSDODNS$ ,

where *DYRR@USECON*: Loan Charge-Off Rate: Residential Real Estate Loans: All Comml Banks (SAAR,%); (Source: H8 Release, Federal Reserve Board);

*XL15HOM5@FFUNDS*: Households and nonprofit organizations; home mortgages; liability (Source: Table L.100, Flow of Funds Accounts);

*DYU@USECON* Loan Charge-Off Rate: Consumer Loans: All Insured Comml Banks (SA, %) (Source: H8 Release, Federal Reserve Board);

*HCCSDODNS@FFUNDS*: Households and nonprofit organizations; consumer credit; liability (Source: Table L.100, Flow of Funds Accounts).

Both in the model and in the data, charge-offs rates are scaled by steady-state GDP. In the data, liabilities are in dollars and steady-state GDP is measured by a cubic trend in the sum of nominal consumption and investment.

*Notes:* Charge-offs for mortgages (*DYRR*) are available starting in 1991Q1, whereas charge-offs for Consumer Loans (*DQU*) begin in 1985Q1. I use the regression coefficients of a regression of *DYRR* on a constant and *DQU* for the 1991–2010 period and data on *DYI* in order to backcast the missing data for *DYRR* for the 1986–1990 period.

#### 5. Loans to entrepreneurs

*Model variable:*  $L_{E,t}$ .

*Data:*  $L_{E,t} = OL14MOR5 + OL14OTL5 + OL14BLN5$ . The series is converted in real terms using the GDP deflator, log transformed and detrended with a quadratic trend.

#### 6. Loans to households

*Model variable:*  $L_{H,t}$ .

*Data:*  $L_{H,t} = XL15HOM5 + HCCSDODNS$ . The series is converted in real terms using the GDP deflator, log transformed and detrended with a quadratic trend.

#### 7. House prices

*Model variable:*  $q_t$ .

*Data:* *USHPI@USECON*: FHFA House Price Index, United States (NSA). The series is converted in real terms using the GDP deflator, log transformed and detrended with a quadratic trend (Source: FHFA).

#### 8. Technology (TFP)

*Model variable:*  $A_{Z,t}$ .

*Data:* Utilization-adjusted quarterly growth rate of TFP (*DTFP\_UTIL@SFFED*) constructed by [Fernald \(2012\)](#). The series is integrated back to levels, log transformed, and detrended with a quadratic trend.

*Notes:* Fernald corrects the Solow residual (a measure of TFP) by utilization (and other adjustments) to arrive at a measure of the growth rate of technology. The utilization-adjusted quarterly series is an improvement over more “naïve” measures of TFP as a high-frequency indicator of technological change”. As shown in the bottom right panel of [Fig. 2](#), it is hard to characterize the behavior of TFP during the financial crisis in simple terms: TFP is weak around the 2005–2008 period, rises in 2009 in the midst of the financial crisis, and drops again around 2010 (by contrast, TFP without the utilization adjustment does not rise in 2009, as utilization drops substantially at the peak of the financial crisis).

### Appendix D. Additional notes on charge-offs

Charge-off rates are the flow of a bank’s net charge-offs (gross charge-offs minus recoveries) during a quarter divided by the average level of its loan outstanding over that quarter multiplied by 400 to express the ratio as an annual percentage rate. Charged-off loans are reported on schedule RI-B and the average levels of loans on schedule RC-K of a bank’s quarterly Consolidated Report of Condition and Income (generally referred to as the call report). Charge-off rates on loans are then computed dividing bank’s net charge-offs by average outstanding loans of banks.

For the purpose of computing total losses of all financial intermediaries, I apply bank charge-off rates to the entire stock of mortgage debt held by households and businesses in the U.S. Note, in fact, that bank loans are only a fraction of total loan payables of households and businesses, since many loans are sold after origination to GSE and secondary market investors. For instance, as shown in Table L.217 of the Flow of Funds data, the total stock of mortgage debt (held by households and businesses) in the U.S. at the end of 2010 was \$13.7tn. Out of this amount, \$4.2tn is held by banks (largely, U.S. chartered depository institutions) which file the call reports, whereas the rest is held by GSEs and Agency- and GSE-backed mortgage pools (\$6.2tn), by ABS issuers (\$2tn), and a smaller fraction by REITs, Finance Companies, Credit Unions. By allocating

all losses to banks, I am effectively consolidating GSE, commercial banks and ABS issuers into one single, big, financial institution. Note also that GSEs may issue liabilities to finance issuance of ABS, and some of their liabilities are in turn owned by banks.

How big were the charge-offs during the financial crisis? If one considers charge-offs at all insured commercial banks, net charge-offs were \$150bn above baseline per year for about 3 years, for a total cumulative loss of around \$450bn. Charge-offs of \$176bn in 2009 against a loan volume of \$6,647bn in the same year (broken down into \$966bn of consumer loans, \$2,099bn of residential real estate loans, and \$1,344bn of commercial real estate loans) indicate a charge-off rate of 2.5 percent, and a ratio of charge-offs to GDP of around 1.5 percent. If one now takes the same charge-off rate but applies it to all debt instruments of households and businesses in the United States, cumulative loan losses in dollars become much larger, since they now apply to a stock of household debt of \$13,394bn in 2009, and a stock of nonfinancial business debt of \$6,416bn. Hence the resulting losses are about \$1.2tn.

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