Technical Appendix: International Business Cycles with Domestic and Foreign Lenders

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1. The model

As the two countries are symmetric, we only describe, without loss of generality, agents' decisions in the domestic economy.

1.1. Domestic entrepreneurs

The production function is Cobb-Douglas in domestically located labor l_t and real estate owned by entrepreneurs:

$$y_t = A_t h_{t-1}^{\nu} l_t^{1-\nu} \tag{1.1}$$

where productivity follows an exogenous stochastic stationary $AR(1)$ process around a constant mean A. Entrepreneurs maximize their lifetime utility from the consumption flow c_t. Denoting with E_t the expectation operator conditional on time t information and with γ the entrepreneurs' discount factor, entrepreneurs solve the following problem:

$$\max_{b_t^H, b_t^F, , h_t, l_t, \alpha_t} E_0 \sum_{t=0}^{\infty} \gamma^t \ln c_t \tag{1.2}$$

subject to flows and borrowing constraints:

$$A_t h_{t-1}^{\nu} l_t^{1-\nu} + b_t^H + b_t^F = c_t + q_t (h_t - h_{t-1}) + R_{t-1}^H b_{t-1}^H + R_{t-1}^F b_{t-1}^F + w_t l_t \tag{1.3}$$

$$R_t^H b_t^H \leq m_H \alpha_t q_{t+1} h_t \tag{1.4}$$

$$R_t^F b_t^F \leq q_{t+1} (1 - \alpha_t) h_t \left(1 - \frac{1 - m_F}{qh} (q_{t+1} (1 - \alpha_t) h_t)\right) \tag{1.5}$$

Entrepreneurs choose labor and real estate; how much to borrow from domestic and foreign households; how to allocate shares α_t of real estate between domestic and foreign financiers.

Define λ_t^H and λ_t^F as the time t shadow values of the domestic and foreign borrowing constraint respectively. The first-order conditions for an optimum are the consumption Euler equations (1.6

and 1.7), real estate demand (1.8), choice of α_t (1.9), and labor demand (1.10):

$$\frac{1}{c_t} = E_t\left(\frac{\gamma R_t^H}{c_{t+1}}\right) + \lambda_t^H R_t^H \tag{1.6}$$

$$\frac{1}{c_t} = E_t\left(\frac{\gamma R_t^F}{c_{t+1}}\right) + \lambda_t^F R_t^F \tag{1.7}$$

$$\frac{1}{c_t} q_t = E_t\left(\begin{array}{c} \frac{\gamma}{c_{t+1}}\left(\nu \frac{y_{t+1}}{h_t} + q_{t+1}\right) + \lambda_t^H m_H \alpha_t q_{t+1} \\ +\lambda_t^F (1-\alpha_t) q_{t+1}\left(1 - \frac{2(1-m_F)(1-\alpha_t)q_{t+1}h_t}{qh}\right) \end{array}\right) \tag{1.8}$$

$$\lambda_t^H m_H = \lambda_t^F E_t\left(1 - \frac{2(1-m_F)(1-\alpha_t) q_{t+1} h_t}{qh}\right) \tag{1.9}$$

$$w_t l_t = (1-\nu) y_t. \tag{1.10}$$

The optimal value of α. Using 1.6, 1.7, and 1.9, we can solve for the optimal α_t as a function of the credit multipliers and of the value of real estate held by entrepreneurs as follows:

$$\alpha_t = 1 - \frac{1 - \frac{\lambda_t^H}{\lambda_t^F} m_H}{1 - m_F} \frac{qh}{2 q_{t+1} h_t} \tag{1.11}$$

so that in steady state

$$\alpha = 1 - \frac{1 - m_H}{2(1 - m_F)}. \tag{1.12}$$

Equation 1.11 shows how in general the optimal value of α, the share of domestic collateral, will be positively related to the average domestic loan-to-value ratio (m_H) and inversely related to average foreign loan-to-value ratio (m_F). In a neighborhood of the steady state, whenever asset values increase, α will increase. That is, asset price increases will be associated with a switch from foreign towards domestic lenders.

Linearizing 1.11 around the steady state, letting $qh = v$, gives:

$$\alpha_t = 1 - \frac{1 - \frac{\lambda_t^H m_H}{\lambda_t^F}}{1 - m_F} \frac{v}{v_t} \tag{1.13}$$

$$\alpha \widehat{\alpha}_t = \frac{1 - \frac{\lambda_t^H m_H}{\lambda_t^F}}{1 - m_F} \frac{v}{v^2} dv_t = \frac{1 - m_H}{1 - m_F} \widehat{v}_t = (1 - \alpha) \widehat{v}_t \tag{1.14}$$

$$\widehat{\alpha}_t = \frac{1 - \alpha}{\alpha} \widehat{v}_t \tag{1.15}$$

so that the linearized borrowing constraint

- for domestic loans is

$$\widehat{R}_t + \widehat{b}_t = \widehat{\alpha}_t + \widehat{v}_t = \frac{1}{\alpha} \widehat{v}_t \tag{1.16}$$

- for foreign loans is, letting $s_t = 1 - \alpha_t$ and $q_t h_t = v_t$

$$
\begin{aligned}
R_t^F b_t^F &= s_t v_t - \frac{(1-m_F)}{v} v_t^2 s_t^2 && (1.17)\\
bdR_t + Rdb_t &= vds_t + sdv_t - \frac{1-m_F}{v}\left(2v^2 sds_t + 2s^2 vdv_t\right) && (1.18)
\end{aligned}
$$

$$
Rb\left(\widehat{R}_t + \widehat{b}_t\right) = sv\left(\widehat{s}_t + \widehat{v}_t\right) - 2\left(1-m_F\right)s\left(sv\widehat{s}_t + s\widehat{v}_t\right) \quad (1.19)
$$

$$
\begin{aligned}
Rb\left(\widehat{R}_t + \widehat{b}_t\right) &= sv\left(\widehat{s}_t + \widehat{v}_t\right) - 2\left(1-m_F\right)s\left(sv\right)\left(\widehat{s}_t + \widehat{v}_t\right) && (1.20)\\
\left(\widehat{R}_t + \widehat{b}_t\right) &= \frac{sv}{Rb}\left(\widehat{s}_t + \widehat{v}_t\right)\left(1 - 2\left(1-m_F\right)s\right) && (1.21)
\end{aligned}
$$

in a neighborhood of the steady state

$$
\begin{aligned}
\widehat{\alpha}_t &= \frac{1-\alpha}{\alpha}\widehat{v}_t && (1.22)\\
s_t &= 1-\alpha_t && (1.23)\\
\widehat{s}_t &= -\frac{\alpha}{s}\widehat{\alpha}_t && (1.24)\\
\widehat{s}_t &= -\frac{\alpha}{\sigma}\frac{1-\alpha}{\alpha}\widehat{v}_t = -\widehat{v}_t && (1.25)
\end{aligned}
$$

Hence

$$
\widehat{R}_t^F + \widehat{b}_t^F = 0 \quad (1.26)
$$

1.2. Domestic households

The household sector (denoted with a prime) is conventional. In each period, household enter with real estate holdings h'_{t-1} (priced at q_t) and bonds coming to maturity. They derive utility from consumption of the final good c'_t and from real estate services proportional to the housing stock h'_t. They rent labor l'_t to domestic entrepreneurs at a wage w_t (labor is immobile across countries), lend b_t^H to domestic firms, b_t^F to foreign firms and lend b_t (or borrow $-b_t$) to (from) foreign households, while receiving back the amount lent in the previous period times the agreed interest rates, respectively R^H, R^F and R.

Formally, households solve:

$$
\max_{b_t^H, b_t^F, h'_t, b_t, l_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln c'_t + j\ln h'_t - \frac{\tau}{\eta} l_t^{\eta}\right) \quad (1.27)
$$

where β is households' discount factor. As we mentioned already, we assume that $\gamma < \beta$. This ensures that entrepreneurs' borrowing constraints will always hold with the equality sign.

The flow of funds for the household is:

$$
c'_t + q_t\left(h'_t - h'_{t-1}\right) + b_t^H + b_t^F + b_t + \psi\frac{\left(b_t - b\right)^2}{2} = R_{t-1}^H b_{t-1}^H + R_{t-1}^F b_{t-1}^F + R_{t-1} b_{t-1} + w_t l_t \quad (1.28)
$$

The $\psi\frac{\left(b_t - \overline{b}\right)^2}{2}$ term reflects the adjustment cost of an increase in the household portfolio holding vis-a-vis the rest of the world

Solving this problem yields (choosing $b_t, b_t^H, b_t^F, l_t, h_t$)

$$\frac{1}{c_t'}\left(1+\psi\left(b_t-\overline{b}\right)\right) = \beta E_t\left(\frac{R_t}{c_{t+1}'}\right) \tag{1.29}$$

$$R_t^H = R_t^F \tag{1.30}$$

$$R_t^H = \frac{R_t}{1+\psi\left(b_t-\overline{b}\right)} \tag{1.31}$$

$$w_t = \tau c_t' l_t^{\eta-1} \tag{1.32}$$

$$\frac{q_t}{c_t'} = \frac{j}{h_t'} + \beta E_t\left(\frac{q_{t+1}}{c_{t+1}'}\right). \tag{1.33}$$

Finally, in a steady state with constant consumption, $\beta R = 1$ and

$$b_t = \overline{b} \tag{1.34}$$

1.3. Foreign economy

The foreign economy is completely symmetric to the domestic one and is therefore omitted. We just report first order conditions for foreign households involving bond adjustment cost. They are given by:

$$\frac{1}{c_t^{*\prime}}\left(1-\psi\left(b_t-\overline{b}\right)\right) = \beta E_t\left(\frac{R_t}{c_{t+1}^{*\prime}}\right) \tag{1.35}$$

$$R_t^{H^*} = R_t^{F^*} \tag{1.36}$$

$$R_t^{H^*} = \frac{R_t}{1-\psi\left(b_t-\overline{b}\right)}. \tag{1.37}$$

where we denote foreign variables with a star *. For symmetry, we assume $m_H = m_H$ and $m_F = m_{F^*}$.

2. Bonds and interest rates

The following table summarizes the range of financial assets that are traded in the economy:

symbol	*bond origination*	*issuer*	*buyer*	*collateral*	*price*
b^H	home	home firm	home household	αh	R^H
b^F	home	home firm	foreign household	$(1-\alpha)\,h$	R^F
b^{H^*}	foreign	foreign firm	home household	$(1-\alpha^*)\,h^*$	R^{H^*}
b^{F^*}	foreign	foreign firm	foreign household	$\alpha^* h^*$	R^{F^*}
b	(foreign)	foreign household	home household	-	R

It is easy but notationally cumbersome to define an equilibrium. Before doing so, notice that in equilibrium household will demand the same return on domestic versus foreign assets:

$$R_t^H = R_t^F = \frac{R_t}{1+\psi\left(b_t-\overline{b}\right)} \tag{2.1}$$

whereas for foreign households the similar arbitrage equation will be:

$$R_t^{H^*} = R_t^{F^*} = \frac{R_t}{1 - \psi\left(b_t - \overline{b}\right)} \tag{2.2}$$

Combine 2.1 and 2.2 to get:

$$R_t^{H*}\left(1 - \psi\left(b_t - \overline{b}\right)\right) = R_t^{F^*}\left(1 - \psi\left(b_t - \overline{b}\right)\right) = R_t = R_t^H\left(1 + \psi\left(b_t - \overline{b}\right)\right) = R_t^F\left(1 + \psi\left(b_t - \overline{b}\right)\right) \tag{2.3}$$

and in log-linear terms: using $\widehat{b}_t = b_t - \overline{b}$:

$$\widehat{R}_t^{H*} - \psi\widehat{b}_t = R_t^{F*} - \psi\widehat{b}_t = R_t = R^H + \psi\widehat{b}_t = R_t^F + \psi\widehat{b}_t. \tag{2.4}$$

Taken together, these two conditions imply that at each date a unique interest rate holds for all bonds if $\psi = 0$.

3. Equilibrium

For given bond holdings $\left(b_{t-1}^H, b_{t-1}^F, b_{t-1}^{H^*}, b_{t-1}^{F^*}, b_{t-1}\right)$, interest rates $\left(R_{t-1}, R_{t-1}^H, R_{t-1}^F, R_{t-1}^{H^*}, R_{t-1}^{F^*}\right)$, domestic real estate holdings $\left(h_{t-1}, h'_{t-1}\right)$, foreign real estate holdings $\left(h_{t-1}^*, h_{t-1}^{\prime *}\right)$, and technology $\left(A_t, A_t^*\right)$, a recursive competitive equilibrium is characterized by:

1. a path of asset prices $\left(q_t, q_t^*\right)$;
2. a path of interest rates $\left(R_t, R_t^H, R_t^F, R_t^{H^*}, R_t^{F^*}\right)$;
3. a path of wage rates $\left(w_t, w_t^*\right)$;
4. consumption $\left(c_t, c'_t, c_t^*, c_t^{'*}\right)$;
5. real estate holdings $\left(h_t, h'_t, h_t^*, h_t^{'*}\right)$;
6. bond holding dynamics $\left(b_t^H, b_t^F, b_t^{H^*}, b_t^{F^*}, b_t\right)$;
7. labour supply $\left(l_t, l_t^*\right)$;
8. output dynamics $\left(y_t, y_t^*\right)$;
9. multipliers on the credit constraints $\left(\lambda_t, \lambda_t^*\right)$;
10. shares of domestic collateral $\left(\alpha_t, \alpha_t^*\right)$;

such that:

1. the bond markets clear;
2. the domestic and foreign labor markets clear;
3. the domestic and foreign real estate markets clear;
4. the world final good market clears.

4. Introducing variable capital

Add now variable capital k as a factor of production. Assuming k is collateralizable as well, we denote z_H the domestic loan to value and σ_t the share of collateral which is pledged to domestic lenders. The two borrowing constraints will now be written as

$$R_t^H b_t^H \leq m_H \alpha_t q_{t+1} h_t + z_H \sigma_t k_t \tag{4.1}$$

$$\begin{aligned} R_t^F b_t^F \leq\; & q_{t+1} \left(1-\alpha_t\right) h_t \left(1 - \left(1-m_F\right) \frac{q_{t+1}\left(1-\alpha_t\right) h_t}{qh}\right) \\ & + \left(1-\sigma_t\right) k_t \left(1 - \left(1-z_F\right) \frac{\left(1-\sigma_t\right) k_t}{2k}\right) \end{aligned} \tag{4.2}$$

The first order conditions for k_t and σ_t are:

$$\frac{1}{c_t} = E_t \left(\frac{\gamma}{c_{t+1}} \left(\mu \frac{y_{t+1}}{k_t} + 1 - \delta\right) + \lambda_t^H z_H \sigma_t + \lambda_t^F \left(1-\sigma_t\right) \left(1 - \frac{\left(1-z_F\right)\left(1-\sigma_t\right) k_t)}{k}\right)\right) \tag{4.3}$$

$$z_H = \frac{\lambda_t^F}{\lambda_t^H} \left(1 - \left(\left(1-z_F\right)\left(1-\sigma_t\right) \frac{k_t}{k}\right)\right) \tag{4.4}$$

Combining the two expression yields:

$$\frac{1}{c_t} = E_t \left(\frac{\gamma}{c_{t+1}} \left(\mu \frac{y_{t+1}}{k_t} + 1 - \delta\right) + \lambda_t^H z_H\right) \tag{4.5}$$

or:

$$\frac{1}{c_t} \left(1 - \frac{z_H}{R_t^H}\right) = E_t \left(\frac{\gamma}{c_{t+1}} \left(\mu \frac{y_{t+1}}{k_t} + 1 - \delta - z_H\right)\right) \tag{4.6}$$

When linearized, the equation above becomes:

$$-\widehat{c}_t + \frac{\beta z_H}{1-\beta z_H} \widehat{R}_t^H = -\widehat{c}_{t+1} + \left(1 - \gamma \frac{1-\delta}{1-\beta z_H} + \frac{\gamma z_H}{1-\beta z_H}\right) \left(\widehat{y}_{t+1} - \widehat{k}_t\right) \tag{4.7}$$

5. The steady state

The steady state is described by the following equations:

$$\alpha = 1 - \frac{1 - m_H}{2\,(1 - m_F)} \tag{5.1}$$

$$\sigma = 1 - \frac{1 - z_H}{2\,(1 - z_F)} \tag{5.2}$$

$$\frac{k}{y} = \frac{\gamma\mu}{1 - \gamma\,(1 - \delta) - (\beta - \gamma)\,z_H} \tag{5.3}$$

$$\frac{qh}{y} = \frac{\gamma\nu}{1 - \gamma - (\beta - \gamma)\,m_H} \tag{5.4}$$

$$\frac{Rb^H}{y} = \alpha m_H \frac{qh}{y} + \sigma z_H \frac{k}{y} \tag{5.5}$$

$$\frac{Rb^F}{y} = (1 - \alpha)\,(1 - (1 - m_F)\,(1 - \alpha))\,\frac{qh}{y} + (1 - \sigma)\,(1 - (1 - z_F)\,(1 - \sigma))\,\frac{k}{y} \tag{5.6}$$

$$\frac{c'}{y} = (1 - \beta)\left(\frac{Rb^H}{y} + \frac{Rb^{F*}}{y}\right) + (1 - \mu - \nu) \tag{5.7}$$

$$\frac{q\,(1 - h)}{c'} = \frac{j}{1 - \beta} \tag{5.8}$$

$$\frac{h}{1 - h} = \frac{\gamma\nu\,(1 - \beta)}{j\,(1 - \gamma - (\beta - \gamma)\,m_H)} \div \frac{c'}{y} \tag{5.9}$$

$$\frac{c}{y} = (\mu + \nu) - \delta\frac{k}{y} - (R - 1)\left(\frac{b^H}{y} + \frac{b^F}{y}\right) \tag{5.10}$$

Solving for b^H and b^F gives:

$$\frac{Rb^H}{y} = \left(1 - \frac{1 - m_H}{2\,(1 - m_F)}\right) m_H \frac{qh}{y} + \left(1 - \frac{1 - z_H}{2\,(1 - z_F)}\right) z_H \frac{k}{y} \tag{5.11}$$

and

$$\frac{Rb^F}{y} = \frac{1 - m_H^2}{4\,(1 - m_F)}\,\frac{qh}{y} + \frac{1 - z_H^2}{4\,(1 - z_F)}\,\frac{k}{y}. \tag{5.12}$$

6. The log-linear equilibrium

6.1. The model with variable capital, endogenous α and no quadratic costs of bond-holdings

Here is our log-linearized model, allowing for variable capital. All the variables with a t subscript are in % deviations from their steady state levels (indicated without subscripts). For intra-household bonds b_t, which are close to zero in steady state, we approximate b_t/y. Normalize steady state labor supply so that income is unity, that is $y = y^* = 1$. Let $\widetilde{\Delta} k_t = k_t - (1-\delta) k_{t-1}$. After some algebra, some variables can be eliminated and the equilibrium can be reduced to a dynamic system of 18 equations in 18 unknowns, where the unknowns are:

$$c \quad c' \quad h \quad k \quad y \quad c^* \quad c^{*\prime} \quad h^* \quad k^* \quad y^* \quad b \quad b^H \quad b^F \quad b^{H*} \quad b^{F*} \quad R \quad q \quad q^*$$

and A and A^* follows $AR(1)$ the stochastic processes described in Table 1 of the paper.

The equations for home economy are given by:

$$c'c'_t + b^H b^H_t + b^{F*} b^{F*}_t + b_t = R\left(b^H + b^*\right) R_{-1} + Rb_{-1} + Rb^H b^H_{-1} + Rb^{F*} b^{F*}_{-1} + (1-\mu-\nu) y_t + qh\Delta h_t \tag{6.1}$$

$$(\mu+\nu) y_t + b^H b^H_t + b^F b^F_t = R\left(b^H + b^F\right) R_{-1} + Rb^H b^H_{-1} + Rb^F b^F_{-1} + qh\Delta h_t + cc_t + k\widetilde{\Delta} k_t \tag{6.2}$$

$$c'_{t+1} = c'_t + R_t \tag{6.3}$$

$$q_t = \beta q_{t+1} + \frac{(1-\beta) h}{1-h} h_t + c'_t - \beta c'_{t+1} \tag{6.4}$$

$$\left(1 - (1-\nu-\mu)/\eta\right) y_t = A_t + \nu h_{t-1} + \mu k_{t-1} - \left((1-\nu-\mu)/\eta\right) c'_t \tag{6.5}$$

$$R_t + b^H_t = (1/\alpha) \frac{m_H qh}{m_H qh + z_H k} (q_{t+1} + h_t) + (1/\sigma) \frac{z_H k}{m_H qh + z_H k} k_t \tag{6.6}$$

$$R_t + b^F_t = 0 \tag{6.7}$$

$$q_t - (\gamma + m_H(\beta-\gamma)) q_{t+1} = (1-\beta m_H)(c_t - c_{t+1}) + (1 - \gamma + m_H(\beta-\gamma))(y_{t+1} - h_t) - m_H \beta R_t \tag{6.8}$$

$$0 = (1-\beta z_H)(c_t - c_{t+1}) + (1 - \gamma(1-\delta) - z_H(\beta-\gamma))(y_{t+1} - k_t) - \beta z_H R_t. \tag{6.9}$$

The equations for the foreign economy are given by:

$$c^{*\prime}c_t^{*\prime} + b^{H*}b_t^{H*} + b^F b_t^F - b_t = R\left(b^{H*} + b^F\right)R_{-1} - Rb_{-1} + \tag{6.10}$$
$$+Rb^{H*}b_{-1}^{H*} + Rb^F b_{-1}^F + (1-\mu-\nu)\,y_t^* + q^*h^*\Delta h_t^*$$

$$(\mu+\nu)\,y_t^* + b^{H*}b_t^{H*} + b^{F*}b_t^F = R\left(b^{H*} + b^{F*}\right)R_{-1} + \tag{6.11}$$
$$+Rb^{H*}b_{-1}^{H*} + Rb^{F*}b_{-1}^{F*} + q^*h^*\Delta h_t^* + c^*c_t^* + k^*\widetilde{\Delta}k_t^*$$

$$c_{t+1}^{*\prime} = c_t^{*\prime} + R_t \tag{6.12}$$

$$q_t^* = \beta q_{t+1}^* + \frac{(1-\beta)\,h^*}{1-h^*}h_t^* + c_t^{\prime*} - \beta c_{t+1}^{\prime*} \tag{6.13}$$

$$(1-(1-\nu-\mu)/\eta)\,y_t^* = A_t^* + \nu h_{t-1}^* + \mu k_{t-1}^* - ((1-\nu-\mu)/\eta)\,c_t^{\prime*} \tag{6.14}$$

$$R_t + b_t^{H*} = (1/\alpha^*)\frac{m_H q^* h^*}{m_H q^* h^* + z_{H^*}k^*}\left(q_{t+1}^* + h_t^*\right) + (1/\sigma^*)\frac{z_{H^*}k^*}{m_H q^* h^* + z_{H^*}k^*}k_t^* \tag{6.15}$$

$$R_t + b_t^{F*} = 0 \tag{6.16}$$

$$q_t^* - (\gamma + m_H(\beta-\gamma))\,q_{t+1}^* = (1-\beta m_H)\left(c_t^* - c_{t+1}^*\right) + (1-\gamma+m_H(\beta-\gamma))\left(y_{t+1}^* - h_t^*\right) - m_H\beta R_t \tag{6.17}$$

$$0 = (1-\beta z_{H^*})\left(c_t^* - c_{t+1}^*\right) + (1-\gamma(1-\delta) - z_{H^*}(\beta-\gamma))\,(y_{t+1} - k_t) - \beta z_{H^*}R_t. \tag{6.18}$$

6.2. The economy with quadratic costs of adjusting bonds

In the bond adjustment cost model, remember the log-linear relationship between interest rates:

$$R_t^{H*} - \psi b_t = R_t^{F*} - \psi b_t = R_t = R_t^H + \psi b_t = R_t^F + \psi b_t.$$

The most convenient normalization involves expressing all interest rates as a function of the intra-household rate. Hence we have the following no-arbitrage relationships involving interest rates:

$$\begin{aligned} R_t^H &= R_t - \psi b_t \\ R_t^F &= R_t - \psi b_t \\ R_t^{H^*} &= R_t + \psi b_t \\ R_t^{F^*} &= R_t + \psi b_t. \end{aligned}$$

Hence the following equations change:

$$c'c_t' + b^H b_t^H + b^{F*}b_t^{F*} + b_t = R\left(b^H + b^{F*} + b\right)R_{-1} + Rb_{-1} + Rb^H b_{-1}^H + \tag{5.1'}$$
$$+Rb^{F*}b_{-1}^{F*} + s_y y_t + qh\Delta h_t - \mathtt{Rb}^H\mathtt{fb}_{t-1} + \mathtt{Rb}^{F^*}\mathtt{fb}_{t-1}$$

$$(\mu+\nu)\,y_t + b^H b_t^H + b^F b_t^F = R\left(b^H + b^F\right)R_{-1} + Rb^H b_{-1}^H + Rb^F b_{-1}^F + \tag{5.2'}$$
$$+ qh\Delta h_t + cc_t + k\Delta k_t - \mathtt{Rb}^H\mathtt{fb}_{t-1} - \mathtt{Rb}^F\mathtt{fb}_{t-1}$$

$$c_{t+1}^{\prime} = c_t' + R_t - \mathtt{fb}_t \tag{5.3'}$$

$$R_t + b_t^H - \mathtt{fb}_t = (1/\alpha)\frac{m_H qh}{m_H qh + z_H k}(q_{t+1} + h_t) + (1/\sigma)\frac{z_H k}{m_H qh + z_H k}k_t \tag{5.6'}$$

$$R_t + b_t^F - \mathtt{fb}_t = 0 \tag{5.7'}$$

$$\begin{aligned} q_t - (\gamma + m_H(\beta-\gamma))\, q_{t+1} &= (1-\beta m_H)(c_t - c_{t+1}) + \\ &+ (1-\gamma+m_H(\beta-\gamma))(y_{t+1}-h_t) - m_H\beta R_t + \mathtt{m}_H\beta\mathtt{fb}_t \end{aligned} \tag{5.8'}$$

$$0 = (1-\beta z_H)(c_t - c_{t+1}) + (1-\gamma(1-\delta) - z_H(\beta-\gamma))(y_{t+1}-k_t) - \beta z_H R_t + \beta\mathtt{z}_H\mathtt{fb}_t. \tag{5.9'}$$

The modifications for the foreign economy are:

$$\begin{aligned} c^{*\prime}c_t^{*\prime} + b^{H*}b_t^{H*} + b^F b_t^F - b_t &= R\left(b^{H*} + b^F - b\right) R_{-1} - Rb_{-1} + \\ &+ Rb^{H*}b_{-1}^{H*} + Rb^F b_{-1}^F + s_y y_t^* + q^* h^* \Delta h_t^* + \\ &+ \mathtt{Rb}^{H^*}\mathtt{fb}_{t-1} - \mathtt{Rb}^F\mathtt{fb}_{t-1} \end{aligned} \tag{5.10'}$$

$$\begin{aligned} (\mu+\nu)\, y_t^* + b^{H*}b_t^{H*} + b^{F*}b_t^F &= R\left(b^{H*} + b^{F*}\right) R_{-1} + Rb^{H*}b_{-1}^{H*} + Rb^{F*}b_{-1}^{F*} + \\ & q^* h^* \Delta h_t^* + c^* c_t^* + k^* \Delta k_t^* + \mathtt{Rb}^{H^*}\mathtt{fb}_{t-1} + \mathtt{Rb}^{F^*}\mathtt{fb}_{t-1} \end{aligned} \tag{5.11'}$$

$$c_{t+1}^{*\prime} = c_t^{*\prime} + R_t + \mathtt{fb}_t \tag{5.12'}$$

$$R_t + b_t^{H*} + \mathtt{fb}_t = (1/\alpha^*)\frac{m_H q^* h^*}{m_H q^* h^* + z_{H^*}k^*}\left(q_{t+1}^* + h_t^*\right) + (1/\sigma^*)\frac{z_{H^*}k^*}{m_H q^* h^* + z_{H^*}k^*}k_t^* \tag{5.15'}$$

$$R_t + b_t^{F*} + \mathtt{fb}_t = 0 \tag{5.16'}$$

$$\begin{aligned} q_t^* - (\gamma + m_H(\beta-\gamma))\, q_{t+1}^* = (1-\beta m_H)\left(c_t^* - c_{t+1}^*\right) + \\ + (1-\gamma+m_H(\beta-\gamma))\left(y_{t+1}^* - h_t^*\right) - m_H\beta R_t - \mathtt{m}_H\beta\mathtt{fb}_t \end{aligned} \tag{5.17'}$$

$$\begin{aligned} 0 = (1-\beta z_{H^*})\left(c_t^* - c_{t+1}^*\right) + \\ (1-\gamma(1-\delta) - z_{H^*}(\beta-\gamma))(y_{t+1}-k_t) - \beta z_{H^*}R_t - \beta\mathtt{z}_H\mathtt{fb}_t. \end{aligned} \tag{5.18'}$$

6.3. The economy with exogenous α

In the exogenous-α economy, the linearized borrowing constraints (equations 5.6, 5.7, 5.15 and 5.16) become:

$$R_t + b_t^H = \frac{m_H qh}{m_H qh + z_H k}(q_{t+1} + h_t) + \frac{z_H k}{m_H qh + z_H k}k_t \tag{5.6''}$$

$$R_t + b_t^F = \frac{m_H qh}{m_H qh + z_H k}(q_{t+1} + h_t) + \frac{z_H k}{m_H qh + z_H k}k_t \tag{5.7''}$$

$$R_t + b_t^{H*} = \frac{m_{H^*} qh}{m_{H^*} qh + z_{H^*} k}(q_{t+1} + h_t) + \frac{z_{H^*} k}{m_{H^*} qh + z_{H^*} k}k_t \tag{5.15''}$$

$$R_t + b_t^{F*} = \frac{m_{H^*} qh}{m_{H^*} qh + z_{H^*} k}(q_{t+1} + h_t) + \frac{z^{K^*} k}{m_{H^*} qh + z_{H^*} k}k_t. \tag{5.16''}$$