Technical Appendix for: Input and Output Inventories in General Equilibrium

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Appendix A: Summary of the Model Equations

The model equations

Below, we summarize the equations that describe the equilibrium in our model:

$$Y_{gt} = C_{gt} + K_{gt} - \left(1 - \delta_{K_t^g}\right) K_{-1}^g + K_{st} - \left(1 - \delta_{K_t^s}\right) K_{-1}^s + F_t - (1 - \delta_F) F_{-1} + M_t - (1 - \delta_M) M_{-1} + AC_t \tag{1}$$

$$\varepsilon_{\beta t} (1 - \gamma \varepsilon_{\gamma t}) \frac{W_t^\phi}{Y_{st}^{1+\phi}} = \omega_t \tag{2}$$

$$\varepsilon_{\beta t} \varepsilon_{\gamma t} \varepsilon_{Ft} \gamma \alpha \frac{W_t^\phi X_t^{\mu - \phi}}{C_{gt}^{1+\mu}} = \lambda_t \tag{3}$$

$$\tau \varepsilon_{\beta t} = \lambda_t (1 - \theta_g) \frac{Y_{gt}}{L_{gt}} \tag{4}$$

$$\tau \varepsilon_{\beta t} = \omega_t (1 - \theta_s) \frac{Y_{st}}{L_{st}} \tag{5}$$

$$\lambda_t \left(1 + \frac{\psi_{Kg}}{\delta_{Kg}} \frac{\Delta K_{gt}}{K_{t-1}}\right) = \beta \lambda_{t+1} \left(1 + \frac{\psi_{Kg}}{2\delta_{Kg}} \frac{K_{gt+1}^2 - K_{gt}^2}{K_{gt}^2} - \delta_{Kgt+1} + \theta_g \sigma \frac{Y_{gt+1} H_{t+1}^\nu}{z_{Kgt+1}^\nu K_{gt}^{1+\nu}}\right) \tag{6}$$

$$\lambda_t \left(1 + \frac{\psi_{Ks}}{\delta_{Ks}} \frac{\Delta K_{st}}{K_{st-1}}\right) = \beta \left(\lambda_{t+1} \left(1 + \frac{\psi_{Ks}}{2\delta_{Ks}} \frac{K_{st+1}^2 - K_{st}^2}{K_{st}^2} - \delta_{Kst+1}\right) + \omega_{t+1} \theta_s \frac{Y_{st+1}}{K_{st}}\right) \tag{7}$$

$$\lambda_t \left(1 + \frac{\psi_M}{\delta_M} \frac{\Delta M_t}{M_{t-1}}\right) = \beta \lambda_{t+1} \left(1 + \frac{\psi_M}{2\delta_M} \frac{M_{t+1}^2 - M_t^2}{M_t^2} - \delta_M + \theta_g (1 - \sigma) \frac{Y_{gt+1} H_{t+1}^\nu}{\varepsilon_{Mt+1}^\nu M_t^{1+\nu}}\right) \tag{8}$$

$$\lambda_t \left(1 + \frac{\psi_F}{\delta_F} \frac{\Delta F_t}{F_{t-1}}\right) = \beta \left(\varepsilon_{\beta t+1} \varepsilon_{\gamma t+1} \gamma (1 - \alpha \varepsilon_{Ft+1}) \frac{W_{t+1}^\phi X_{t+1}^{\mu - \phi}}{F_t^{1+\mu}} + \lambda_{t+1} \left(1 + \frac{\psi_F}{2\delta_F} \frac{F_{t+1}^2 - F_t^2}{F_t^2} - \delta_F\right)\right) \tag{9}$$

$$\theta_g \frac{\sigma Y_{gt} H_t^\nu}{z_{Kgt}^{1+\nu} K_{gt}^\nu} = b_{Kg} \left(\zeta_{Kg} z_{Kgt} + 1 - \zeta_{Kg}\right) K_{gt-1} \tag{10}$$

$$\omega_t \theta_s \frac{Y_{st}}{z_{Kst}} = b_{Ks} (\zeta_{Ks} z_{Kst} + 1 - \zeta_{Ks}) K_{st-1} \lambda_t \tag{11}$$

$$\delta_{Kgt} = \delta_{Kg} + b_{Kg} \zeta_{Kg} z_{Kgt}^2 / 2 + b_{Kg} \left(1 - \zeta_{Kg}\right) z_{Kgt} + b_{Kg} \left(\zeta_{Kg}/2 - 1\right) \tag{12}$$

$$\delta_{Kst} = \delta_{Ks} + b_{Ks} \zeta_{Ks} z_{Kst}^2 / 2 + b_{Ks} (1 - \zeta_{Ks}) z_{Kst} + b_{Ks} (\zeta_{Ks}/2 - 1) \tag{13}$$

$$\xi_{Kgt} = \frac{\psi_{K_g}}{2\delta_{Kg}} \left(\frac{K_{gt} - K_{gt-1}}{K_{gt-1}}\right)^2 K_{gt-1} \tag{14}$$

$$\xi_{Kst} = \frac{\psi_{K_s}}{2\delta_{K_s}} \left(\frac{K_{st} - K_{st-1}}{K_{st-1}}\right)^2 K_{st-1} \tag{15}$$

$$\xi_{Mt} = \frac{\psi_M}{2\delta_M} \left(\frac{M_t - M_{t-1}}{M_{t-1}}\right)^2 M_{t-1} \tag{16}$$

$$\xi_{Ft} = \frac{\psi_F}{2\delta_F} \left(\frac{F_t - F_{t-1}}{F_{t-1}}\right)^2 F_{t-1} \tag{17}$$

$$Y_{gt} = (A_{gt} L_{gt})^{1-\theta_g} H_t^{\theta_g} \tag{18}$$

$$H_t = \left(\sigma (z_{gt} K_{gt-1})^{-\nu} + (1 - \sigma) (\varepsilon_{Mt} M_{t-1})^{-\nu}\right)^{-1/\nu} \tag{19}$$

$$X_t = \left(\alpha \varepsilon_{Ft}^{-\mu} C_t^{-\mu} + (1 - \alpha) F_{t-1}^{-\mu}\right)^{-1/\mu} \tag{20}$$

$$W_t = \left(\gamma \varepsilon_{\gamma t} X_t^{-\phi} + (1 - \gamma \varepsilon_{\gamma t}) \varepsilon_{Ft}^{-\phi} Y_{st}^{-\phi}\right)^{-1/\phi} \tag{21}$$

$$Y_{st} = (A_{st} L_{st})^{1-\theta_s} (z_{st} K_{st-1})^{\theta_s}. \tag{22}$$

The stochastic processes for the shocks are described in the main text. The term AC_t in the first equation denotes the total adjustment costs. The term λ_t denotes the marginal utility of one unit of goods' wealth. The term ω_t determine the marginal utility of one unit of services. We can thus interpret λ_t/ω_t as the price of goods in terms of services.

The steady state

The first-order conditions for fixed capital in the goods sector and input inventories imply that in the steady state, the capital-to-output ratio in the goods sector, $k_g = K_g/Y_g$, and the input-inventories-to-output ratio, $m = M_g/Y_g$, can be written as

$$k_g = \frac{\theta_g \sigma \beta}{1-\beta(1-\delta_{Kg})} \frac{1}{\sigma + (1-\sigma)\left(\frac{\sigma}{1-\sigma}\frac{1-\beta(1-\delta_M)}{1-\beta(1-\delta_{Kg})}\right)^{\frac{\nu}{1+\nu}}} \tag{23}$$

$$m = \frac{\theta_g \beta(1-\sigma)}{1-\beta(1-\delta_M)} \frac{1}{(1-\sigma) + \sigma\left(\frac{\sigma}{1-\sigma}\frac{1-\beta(1-\delta_M)}{1-\beta(1-\delta_{Kg})}\right)^{-\frac{\nu}{1+\nu}}} . \tag{24}$$

These conditions state that the capital-to-output ratio and the input-inventory-to-output ratio are increasing in their relative weights in production, σ and $1-\sigma$, respectively. At the same time, the different factor intensities depend on the degree of substitutability. When the ratios in large parentheses are larger than one (a condition that holds in the data since input inventories are much smaller than capital), then capital is decreasing in ν and input inventories are increasing in ν.

The optimality conditions for goods consumption and output inventories imply:

$$c_g = \left(\frac{\alpha}{1-\alpha}\frac{1-\beta(1-\delta_F)}{\beta}\right)^{\frac{1}{1+\mu}} f , \tag{25}$$

where $c_g = C_g/Y_g$ and $f = F/Y_g$. The ratio of consumption to output inventories is increasing in α, while it is decreasing (increasing) in μ when the term in parentheses is larger (smaller) than one. Using the linear homogeneity of the CES aggregators and the first-order conditions for K_s, C_g, and C_s, we derive the following expression for $k_s = K_s/Y_g$:

$$k_s = \left(\frac{\lambda}{\omega}\right)^{\frac{-\phi}{1+\phi}} \left(\frac{1-\gamma}{\alpha\gamma}\right)^{\frac{1}{1+\phi}} \frac{\theta_s \beta}{1-\beta(1-\delta_{Ks})} \left(\alpha + (1-\alpha)\left(\frac{c_g}{f}\right)^{\mu}\right)^{\frac{\mu-\phi}{\mu(1+\phi)}} c_g. \tag{26}$$

This equation says that capital in the services sector is higher when the relative price of goods in terms of services (λ/ω) is low, when the weight to services in utility, $1-\gamma$, is high, or when the production function for services is capital intensive (θ_s high). Using the first-order conditions for labor and the linear homogeneity of the production functions, the relative price of goods is:

$$\frac{\lambda}{\omega} = \left(\frac{(1-\theta_s)\left(\frac{\theta_s\beta}{1-\beta(1-\delta_{Ks})}\right)^{\frac{\theta_s}{1-\theta_s}}}{(1-\theta_g)\left(\sigma k_g^{-\nu} + (1-\sigma)m_g^{-\nu}\right)^{\frac{-\theta_g}{\nu(1-\theta_g)}}}\right)^{1-\theta_s} . \tag{27}$$

Finally, the market-clearing condition for the goods sector is

$$c_g + \delta_F f + \delta_{Kg} k_g + \delta_{Ks} k_s + \delta_M m = 1 . \tag{28}$$

For given parameter values, equations 23 to 28 above can be jointly solved for f, c_g, k_s, k_g, m and λ/ω. The first-order conditions for L_g, L_s, C_g, and Y_s, together with the production functions, can be solved for $L_g, L_s, Y_g, Y_s, \omega$, and λ. (Details for all the derivations are given below.)

Matching steady-state ratios through choices of $\alpha, \theta_g, \theta_s, \sigma$, and γ

For each estimated value of ν, ϕ, μ, δ_F, and δ_M, and given calibrated values for β, δ_{Kg}, and δ_{Ks}, our estimation procedure aims at exactly matching the following steady state ratios that we take to be the average values obtained from the data (denoted with a bar):

$$\begin{aligned}
\overline{f} &= \text{output inventories over goods output} \\
\overline{m} &= \text{input inventories over goods output} \\
\overline{k_g} &= \text{capital stock in goods industries over goods output} \\
\overline{k_s} &= \text{capital stock in service industries over goods output} \\
\overline{y_s'} &= \text{services output over goods output.}
\end{aligned}$$

where $y_s' = \frac{\omega Y_s}{\lambda Y_g} = \frac{\omega}{\lambda} y_s$ measures services output in units of goods output.

Given β, δ_{Ks}, δ_{Ks}, ϕ, μ, ν, δ_F, and δ_M, simple algebra shows that there is a unique set of values of $\alpha, \theta_g, \theta_s, \sigma, \gamma$ that satisfies the five ratios above. These values are obtained as follows. Given the $\left(\overline{k_g}/\overline{m}\right)$ ratios, we obtain

$$\sigma = \frac{\left(\overline{k_g}/\overline{m}\right)^{1+\nu} \frac{1-\beta\left(1-\delta_{K_g}\right)}{1-\beta(1-\delta_M)}}{1 + \left(\overline{k_g}/\overline{m}\right)^{1+\nu} \frac{1-\beta(1-\delta_{Kg})}{1-\beta(1-\delta_M)}} . \tag{29}$$

From the $\overline{c}_g/\overline{f}$ ratio, we derive

$$\alpha = \frac{\left(\overline{c_g}/\overline{f}\right)^{1+\mu} \frac{\beta}{1-\beta(1-\delta_F)}}{1 + \left(\overline{c_g}/\overline{f}\right)^{1+\mu} \frac{\beta}{1-\beta(1-\delta_F)}} . \tag{30}$$

The formula for k_g/y_g can be used to derive

$$\theta_g = \frac{\left(\sigma + (1-\sigma)\left(\overline{k_g}/\overline{m}\right)^{\nu}\right)\left(1-\beta\left(1-\delta_{Kg}\right)\right)}{\sigma\beta} \overline{k_g} . \tag{31}$$

Finally, we need to choose γ and θ_s to match the values of k_s and y_s'. From the formula for k_s/y_s' , we derive an expression for θ_s

$$\theta_s = \frac{1-\beta\left(1-\delta_{Ks}\right)}{\beta} \frac{\overline{k_s}}{\overline{y_s'}} . \tag{32}$$

Last, we need to obtain γ. Using the expressions for k_s/f and k_s/y_s' above, we obtain

$$\gamma = \frac{\left(\overline{c_g}/\overline{y_s'}\right)^{1+\phi} (\omega/\lambda)^{\phi} c_g^{\mu-\phi} \left(\alpha\overline{c}_g^{-\mu} + (1-\alpha)\overline{f}^{-\mu}\right)^{\frac{\mu-\phi}{\mu}}}{\alpha + \left(\overline{c_g}/\overline{y_s'}\right)^{1+\phi} (\omega/\lambda)^{\phi} c_g^{\mu-\phi} \left(\alpha\overline{c}_g^{-\mu} + (1-\alpha)\overline{f}^{-\mu}\right)^{\frac{\mu-\phi}{\mu}}} , \tag{33}$$

where ω/λ can be calculated from expression 27 above.

Summarizing: (1) for given observed values in the data of k_g, k_s, m, c_g, f, and y_s'; and (2) for any possible combination of $(\beta,\ \delta_{Ks},\ \delta_{Ks},\ \phi,\ \mu,\ \nu,\ \delta_F,\ \text{and } \delta_M)$, the values of α, θ_g, θ_s, σ, and γ that satisfy expressions 29 to 33 are consistent with the the steady-state values of the ratios k_g, k_s, m, c_g, f, and y_s'.

Calculating steady-state hours, output, and prices

The optimal labor supply schedules satisfy

$$\tau = \lambda\left(1-\theta_g\right)\frac{Y_g}{L_g} \tag{34}$$

$$\tau = \omega\left(1-\theta_s\right)\frac{Y_s}{L_s}. \tag{35}$$

$>$From the first-order conditions for C_g and Y_S, after some algebra, we obtain the following formula:

$$\lambda Y_g = \frac{\gamma\alpha}{c_g}\left(\gamma\left(\alpha+(1-\alpha)\left(\frac{c_g}{f}\right)^{\mu}\right)+(1-\gamma)\left(\alpha+(1-\alpha)\left(\frac{c_g}{f}\right)^{\mu}\right)^{\frac{\mu-\phi}{\mu(1+\phi)}}\left(\frac{\omega}{\lambda}\frac{\alpha\gamma}{1-\gamma}\right)^{\frac{\phi}{1+\phi}}\right)^{-1}. \tag{36}$$

By the same token, we find that:

$$\omega Y_s = (1-\gamma)\left(\gamma\left(\alpha+(1-\alpha)\left(\frac{c_g}{f}\right)^{\mu}\right)^{\frac{1+\mu}{\mu}\frac{\phi}{1+\phi}}\left(\frac{\omega}{\lambda}\frac{\alpha\gamma}{1-\gamma}\right)^{\frac{-\phi}{1+\phi}}+(1-\gamma)\right)^{-1}. \tag{37}$$

From the production functions, we know that:

$$Y_g = L_g\left(\sigma k_g^{-\nu}+(1-\sigma)\,m_g^{-\nu}\right)^{-\frac{\theta_g}{\nu(1-\theta_g)}} \tag{38}$$

$$Y_s = L_s\left(\frac{K_s}{Y_s}\right)^{\frac{\theta_s}{1-\theta_s}}. \tag{39}$$

Equations 34 through 39 can be then be solved for L_g, L_s, Y_g, Y_s, ω, and λ using a non-linear equation solver.

Appendix B: Data

Most of the data come from the national income and product accounts (NIPA) produced by the U.S. Bureau of Economic Analysis (BEA) and obtained from Haver Analytics. All NIPA data are quarterly and real data are in chain-weighted \$2000. Table B.1 lists the variable names, Haver mnemonics, and variable descriptions. Our model and data exclude government spending. We used the Tornquist approximation for chain-weighted data when constructing the actual model-consistent data, as recommended by Whelan (2002). For simplicity, the formulas in this appendix abstract from the notational details associated with chain weighting.

The NIPA data classify output by sectors called goods (g), structures (t), and services (s):

$$Y = Y_g + Y_t + Y_s.$$

In contrast, inventory investment, ΔV, is classified by industry (goods inventories include the agriculture, mining, and manufacturing industries; structures inventories include the construction industry; and the services sector includes utilities and trade). Thus, the NIPA output and inventory data do not correspond to the inventory-based sectors of our model definitions of goods and services.

To obtain model-consistent data, we condense the three NIPA sectors into two by redefining and combining the NIPA sector variables as follows. First, write the components of private domestic aggregate output as

$$Y = (C_g + I_g + \Delta V_g + NX_g) + (I_t + \Delta V_t) + (C_{sg} + C_{ss} + I_s + \Delta V_s + NX_s) \ ,$$

where $NX = X - IM$ is net exports and government spending is excluded. Household consumption of structures (C_t) does not exist because construction of residential structures is investment, which we assume is installed in the services sector. Household consumption of services, $C_s = C_{sg} + C_{ss}$, includes two components, distinguished by a second subscript indicating the appropriate model sector to which the services consumption data should belong. Thus, C_{sg} represents the consumption of services from industries that distribute goods (utilities and trade), which we redefine as goods consumption. Also, C_{ss} includes the service flow from housing.

Given these definitions, model-consistent goods output is

$$Y_g = (C_g + C_{sg}) + (I_g + I_t + I_s + NX_g + NX_s) + (\Delta V_g + \Delta V_t + \Delta V_s) \ ,$$

and model-consistent services output as

$$Y_s = C_{ss}.$$

Following Cooley and Prescott (1995), we include net exports in investment because there is no foreign sector in the model; instead, net exports are viewed as the net claims of foreigners on the domestic capital stock. The remainder of this appendix explains how each of the relevant variables is defined and constructed.

Consumption

NIPA consumption data are classified by the type of good consumed by households:

$$C = C_{gn} + C_{gd} + C_s \ .$$

In this equation, goods consumption includes nondurables (gn) plus durables (gd); consumption of services (s) includes the service flow obtained from housing. Theoretically, it would be preferable to construct the service flow from other consumer durable goods besides housing, rather than use actual expenditures, but this is not done in the NIPA data (except for automobile leasing, which is implicitly a service yield). Because we are ultimately trying to explain the volatility, and the change in volatility, of actual GDP data, we use the raw NIPA data instead.

To construct model-consistent consumption data, we must reclassify a portion of the NIPA services consumption data (C_{sg}) as goods consumption because the industries associated with those services are in the model's goods sector. The NIPA consumption data treat energy consumption (such as electricity) as a service. However, because this household energy service is output attributed to the utilities industry, which holds

Variable	Mnemonic	Description
C	C	Consumption
C_{gn}	CN	Consumption of nondurable goods
C_{gd}	CD	Consumption of durable goods
C_s	CS	Consumption of services
C_{se}	CSE	Consumption of energy services
I	F	Fixed investment, total
I_r	FR	Fixed investment, residential
IM	M	Imports
P	JC	Consumption chain-weighted price index
P_s	JCS	Consumption of services chain-weighted price index
P_{se}	JCSE	Consumption of energy services chain-weighted price index
V_{ga}	SF	Farm inventories
V_{gm}	SNM	Manufacturing inventories
V_{sw}	SNW	Wholesale trade inventories
V_{sr}	SNR	Retail trade inventories
$V_{SIC,o}$	SNO2	Other inventories, SIC (fixed-weight $1996)
V_{MUC}	SNB	Mining, utilities, and construction inventories
$V_{NAICS,o}$	SNT	Other inventories, NAICS
V_{CW}	RES513	Inventory chain-weighted residual
X	X	Exports

Table B.1: Variable Names and Data Definitions

Note: These Haver mnemonics are for the nominal data; the real data have an 'H' added at the end and, unless otherwise noted, are in chain-weighted $2000.

inventories, it belongs in the model's goods sector.[1] So, we must define household energy (e) consumption services as model-consistent goods consumption, $C_{sg} = C_{sge}$.

Thus, model-consistent data (denoted by a double tilde) for domestic services consumption are

$$\widetilde{\widetilde{C_s}} = C_s - C_{sg} \ ,$$

and model-consistent data for domestic goods consumption are

$$\widetilde{\widetilde{C_g}} = C_{gn} + C_{gd} + C_{sg} \ .$$

Because the underlying NIPA data are based on the type of good consumed, C_{gn} and C_{gd} already contain the output of the retail-trade industry and any output of the wholesale-trade industry that is classified as consumption (i.e., a final sale to consumers rather than an intermediate input into retail trade or manufacturing).[2]

Investment

Capital is a good, so it follows that investment is output of the goods sector. However, our model has two sectors that each accumulates a sector-specific capital stock, so the model requires classification of investment data by the type of sector (or industry) in which the capital is installed. Although the NIPA data do not classify investment by the sector in which it is installed, the BEA provides other annual data source on investment by

[1] We assume that all types of energy are measurable goods distributed (a task of the model goods sector) to consumers. In this regard, electric and natural gas utility firms are similar to firms specializing in wholesale and retail trade, which distribute finished goods from their producers to their final consumers.

[2] One way to think of the different types of "goods" is in terms of their depreciation rates: $0 < \delta_{sh} < \delta_d < \delta_n < \delta_{so} = 1$, where subscript sh denotes housing services and so denotes other services (that is, not a flow from a durable stock).

industry that does, and we use this source to divide total fixed non-residential investment into sector-specific domestic investment.[3]

Before dividing investment into sectors, we first add net exports to the sum of nonresidential (n) and residential (r) fixed investment, as advocated by Cooley and Prescott (1995):

$$\widetilde{\widetilde{I}} = I_n + NX + I_r$$

where $NX = NX_g + NX_s$. We are forced by the data to combine the net exports of the two sectors here, rather than keeping net exports of goods and net exports of services separate in constructing I_g and I_s. During the second half the sample, net exports of goods become negative and large in absolute value so that $(I_g + NX_g) < 0$; net exports of services are positive but relatively small. From a theoretical perspective, it may also be preferable to use NX because it is not clear that the net claims of foreigners defined by NX_g and NX_s map directly to the measures of I_g and I_s, or even to the sectoral components of the installed U.S. capital stock. In any case, by including net exports in investment the data reflect the influences of foreign trade (exports and imports), which changed during the data sample, on inventory investment in the econometric model, albeit in a reduced-form manner. Residential investment is netted out before the following calculations because it is classified as capital installed entirely in the services sector, and will be added back in later.

Next, the share of non-residential fixed investment for the goods sector includes data for the seven inventory-holding industries: agriculture, mining, utilities, construction, manufacturing, wholesale trade, and retail trade. Using annual BEA data on investment by industry (denoted by a double hat), the share (ω) of goods investment is

$$\widehat{\widehat{\omega}} = \frac{\widehat{\widehat{I}}_g}{\widehat{\widehat{I}}} \ .$$

The "real estate and rental and leasing" industry, which is classified as a service industry by the BEA, rents and leases capital to the rest of the economy, a practice that has increased in frequency over time and now represents a large fraction of total capital services in production (especially for structures). Because these data do not identify the sector to which the real estate industry leases its capital, we apply the seven-industry share above to partition real estate and rental and leasing industry investment into both of the two sectors. Finally, these annual share data are interpolated to obtain a quarterly frequency.

Using the constructed shares of investment by sector, foreign-trade adjusted goods-sector investment is

$$\widetilde{\widetilde{I}}_g = \widehat{\widehat{\omega}} \left(I_n + NX \right) \ ,$$

and foreign-trade adjusted services-sector investment is

$$\widetilde{\widetilde{I}}_s = \left(1 - \widehat{\widehat{\omega}} \right) \left(I_n + NX \right) + I_r \ .$$

In each case, the interpolated investment share data is applied to the actual quarterly data on domestic, non-residential fixed investment.

Inventories

According to the NIPA definitions, each output sector is associated with at least one inventory-holding industry,

$$\begin{aligned} V_g &= V_{ga} + V_{gn} + V_{gm} \\ V_t &= V_{tc} \\ V_s &= V_{su} + V_{sw} + V_{sr} \ , \end{aligned}$$

with industries defined as agriculture (a), mining (n), manufacturing (m), construction (c), utilities (u), wholesale trade (w), and retail trade (r). Thus, to construct model-consistent inventories, we redefine the goods sector as the holder of all inventories:

$$\widetilde{\widetilde{V}}_g = V_g + V_t + V_s \ ;$$

[3] These data can be obtained from http://bea.gov/bea/dn/FA2004/Index.asp.

as discussed in the main text, the services sector holds no inventories ($\widetilde{\widetilde{V}}_s = 0$) by assumption.[4]

We further divide total inventories into two types,

$$\widetilde{\widetilde{V}}_g = M + F \ ,$$

where M denotes input and F denotes output. Economic theory provides no clear categorical definition of input and output inventories in general equilibrium. We view goods as being produced and distributed along a supply and distribution chain, so one (but not the only) logical definition of output inventories for our model is simply the last link of the chain, which is the retail industry:

$$\widetilde{\widetilde{M}} = V_{sr} \ .$$

In this case, input inventories are

$$\widetilde{F} = V_{ga} + V_{gn} + V_{gm} + V_{tc} + V_{su} + V_{sw} \ .$$

In general, all non-retail inventory stocks can be considered inputs into production along the supply chain. According to the Census of Construction, V_{tc} certainly is input inventory (raw materials) and does not include unsold finished structures. In actuality, some fraction of the remaining stocks may be sold directly to consumers, and hence should be classified as output inventories, but we assume this fraction is small.

To obtain a long time series of inventory data, we combine non-farm stocks constructed under two different industry classifications: SIC (1947–1997) and NAICS (1987–present).[5] At this high level of industry definition, the manufacturing, wholesale, and retail inventory data are generally consistent across industry classification schemes, so we splice these data series without further manipulation. The inventories for all remaining industries ($*$), however, are defined as follows:

$$\begin{aligned} V_{SIC,*} &= V_{SIC,o} \\ V_{NAICS,*} &= V_{MUC} + V_{NAICS,o} + V_{CW} \ , \end{aligned}$$

where o denotes "other" industries in each classification system; MUC denotes mining, utilities, and construction. CW denotes the chain-weighted residual for real data (real data on an SIC basis are in fixed-weight \$1996, and thus have no residual). In splicing the data, we use the SIC stocks through 1997, and then use the growth rates of the NAICS from 1997 on to extend the SIC data.

Unfortunately, unlike the NIPA consumption and investment data, it is impossible to identify separately the foreign and domestic components of NIPA inventory stocks from existing data. Rather than make *ad hoc* adjustments, we use the actual inventory data as published.

Consumption Prices

The prices of goods and services consumption are constructed analogously to the respective quantities of consumption. Let w_{se} be the nominal expenditure weight for energy services, and $w_{\tilde{s}} = (1 - w_{se})$ be the nominal expenditure weight for model-consistent (non-energy) services. Then, having calculated the appropriate Tornquist index on the data, the model-consistent price of services consumption is

$$\widetilde{\widetilde{P}}_s = (1/w_{\tilde{s}})\left[P_s - w_{se}P_{se}\right] \ .$$

Likewise, let $w_{\tilde{\tilde{s}}}$ be the nominal expenditure weight for model-consistent services, and $w_g = (1 - w_{\tilde{\tilde{s}}})$ be the nominal expenditure weight for model-consistent goods. Then the model-consistent price of goods consumption is

$$\widetilde{\widetilde{P}}_g = (1/w_g)\left[P - w_{\tilde{\tilde{s}}}\widetilde{\widetilde{P}}_s\right] \ .$$

[4] The NIPA make this same assumption, equating output and final sales in both the structures and services sectors, and associating all inventory investment with the goods sector.

[5] Farm, or agricultural, inventory stocks on a consistent industry classification are already available for the full sample period (1947–present).

The ratio of the consumption prices,

$$\frac{\widetilde{\widetilde{P_g}}}{\widetilde{\widetilde{P_s}}} = \frac{\lambda}{\omega} \ ,$$

equals the ratio of Lagrange multipliers from the model's first-order conditions.

Appendix C: Another Look at Output Inventories in the Utility Function

In this appendix, we illustrate the result that there exists a functional equivalence between entering output inventories in the utility function and an alternative formulation that motivates output inventories using the argument that they reduce shopping costs. To keep the notation simple, we abstract from endogenous labor supply, from consumption of services and from capital and input inventories, and consider the problem of a planner that has to allocate an exogenous stream of goods Y_t to either consumption or finished good inventories. Consider the following two alternative models.

Problem 1. In the first formulation, the planner's problem is:

$$\max \sum_{t=0}^{\infty} \beta^t V(C_t, F_t) \text{ subject to } Y_t = C_t + F_t - (1-\delta_F) F_{t-1}$$

where utility V depends on the goods purchased C_t and the stock of inventories F_t. This is essentially the barebones formulation and notation of our model in the main text.

Problem 2. In the second formulation, the problem is:

$$\max \sum_{t=0}^{\infty} \beta^t u(G_t) \text{ subject to } Y_t = G_t + \phi(G_t, F_t) + F_t - (1-\delta_F) F_{t-1}$$

where the term in the budget constraint $\phi(G_t, F_t)$ denotes the real resource cost of purchasing goods, which is assumed to be a decreasing function of the amount of output inventories F_t available and an increasing function of the amount of goods consumed G_t.

Following Feenstra (1986) the two problems are equivalent if $V(C_t, F_t) = u(G_t)$, and $C_t = G_t + \phi(G_t, F_t)$. We therefore interpret C_t as gross consumption including both net consumption G_t and shopping costs $\phi(G_t, F_t)$ .Feenstra focused on justifying the inclusion of money in the utility function and to show the condition for equivalence between that model and one in which liquidity costs are introduced in the budget Our cases is isophorphiic to his, once money balances are replaced with finished good inventories.

Consider our momentary utility function and budget constraint in the paper. Abstracting from services, endogenous labor and capital and input inventories, it reads as

$$\log\left(\alpha C^{-\mu} + (1-\alpha) F^{-\mu}\right)^{-1/\mu}$$

subject to

$$Y_t = C_t + F_t - (1-\delta_F) F_{t-1}.$$

This utility function and budget constraint yield the same first-order conditions and equilibrium conditions of a model where momentary utility is given by:

$$\log G_t$$

and the budget constraint is

$$Y_t = G_t + \phi_t + F_t - (1-\delta_F) F_{t-1}.$$

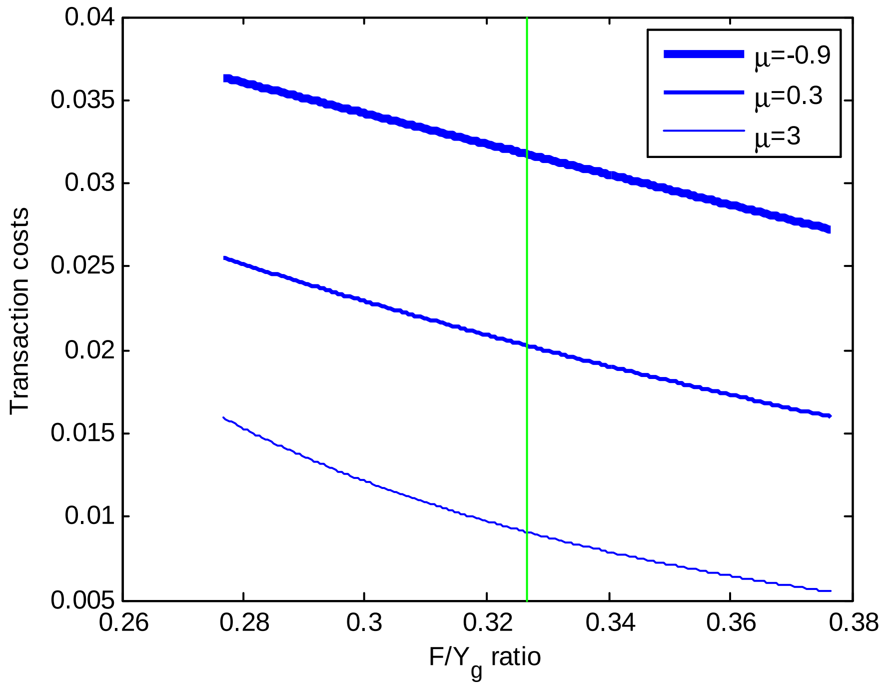
We can find ϕ by setting $G_t = \left(\alpha C_t^{-\mu} + (1-\alpha) F_t^{-\mu}\right)^{-1/\mu}$, solving for C_t and it in $\phi(G_t, F_t) = C_t - G_t$.This yields:

$$\phi(G_t, F_t) = \left(\left(\frac{1}{\alpha} - \frac{1-\alpha}{\alpha}\left(\frac{G_t}{F_t}\right)^{\mu}\right)^{-\frac{1}{\mu}} - 1\right) G_t.$$

One can easily verify that the two problems yield the same first order conditions. The result also holds when one allows for consumption of services and leisure in the utility function, provided they enter utility in an additively separable way, so that the optimality condition with respect to consumption and inventories are not affected. Additive separability of leisure has been assumed from the outset. Approximate additive separability of consumption of services is suggested by our empirical estimates.

Of course, what differs across models is the structural interpretation of the parameters. In our model, α measures the relative weight in utility of goods consumption relative to finished good inventories. In the transaction cost model, α measures the size of the transaction cost: the higher α, the lower the transaction cost (in the limiting case of $\alpha = 1$, there are no transaction costs for purchasing goods, $\phi = 0$ and no inventories are held in equilibrium, just like in our model). In our model, μ measures the elasticity of substitution between G and F. In the transaction cost model, μ measures the way in which more inventories of finished goods reduce or increase the shopping cost of obtaining a given amount of consumption goods. In particular, a higher estimate of μ in the transaction cost model implies: (1) lower average transaction costs on average; (2) a more convex transaction cost function itself.

Below, we plot how our estimates of μ (.3) and α (.9668) map in the resulting ϕ function. We also assume that G/F is at its steady state value and that measured consumption in the data corresponds to net consumption G As can be seen in the figure, the transaction cost function is decreasing (and convex) in F. At our estimates (given the calibrated value for F/Y_g), the function implies transaction costs equal to 2% of total goods output (see the figure below). The other two lines plot the transaction cost function under the assumption that $\mu = -0.9$ and $\mu = 3$ respectively.



Appendix D: A Model of Input Inventories with Usage Only

Here we sketch a version of the model in the paper where we allow for a treatment of input inventories that ignores their convenience service but models explicitly their usage.

Assume that (1) only input inventories that are "used up" augment society's ability to produce more; and that (2) the usage of inventories depends upon the beginning of period stock. This specification leads to a gross production function of the form:

$$Y_{gt} = (A_{gt}L_{gt})^{1-\theta_g}\left(\sigma\left(z_{gt}K_{gt-1}\right)^{-\nu} + (1-\sigma)\left(\varepsilon_{Mt}z_{Mt}M_{t-1}\right)^{-\nu}\right)^{-\theta_g/\nu}$$

where $z_{Mt} \in (0,1)$ is the utilization rate of inventories, so that $m_t = z_{Mt}M_{t-1}$ is the amount of inventories that are used (and used up) in production. We assume that higher utilization of the stock M leads to a higher depreciation in a convex fashion, and that depreciation rate and utilization rate are equal in a neighborhood of the steady state. These considerations lead us to write a "depreciation function" for inventories as follows:

$$d_{Mt} = \delta_M + z_{Mt} + a_{Mt}\left(z_{Mt}\right)$$

where: δ_M is a fixed component of the depreciation rate unrelated to usage and reflecting wastage and/or linear holding costs; z_{Mt} captures the usage of materials (proportional to the stock); and the term a_{Mt} describes the additional component of wastage that depends upon utilization in a convex fashion: this component reflects the idea that, at the margin, a higher or faster usage might provoke collateral damage to the remaining parts of the stock that are not directly used in production. Namely, we assume that $a_{Mt} = R_M\left(\frac{\zeta_M}{2} - 1 + (1-\zeta_M)\frac{z_{Mt}}{z_M} + \frac{\zeta_M}{2}\frac{z_{Mt}^2}{z_M^2}\right)$, where $R_M = \frac{1}{\beta} = 1 - \delta_M$. The function a_{Mt} is convex in z_{Mt} and is normalized so that it equals zero when z_{Mt} equals the optimal, steady-state choice z_M.[6] The assumption of convexity has two appealing properties: first, it allows us to solve the model using standard perturbation methods; second, it captures the idea that, at the margin, a higher utilization rate leads to a higher depreciation. This is reflected in the functional form of a_{Mt}. The good's sector resource constraint is now:

$$\begin{aligned} Y_{gt} \quad = \quad & C_{gt} + K_{gt} - \left(1-\delta_{K_t^g}\right)K_{t-1}^g + K_{st} - \left(1-\delta_{K_t^s}\right)K_{t-1}^s + F_t - (1-\delta_F)F_{-1} \\ & + M_t - \left(1-\delta_M - z_{Mt} - a_{Mt}\left(z_{Mt}\right)\right)M_{-1} + z_{Mt}M_{t-1} + AC_t. \end{aligned} \tag{40}$$

This equations illustrates how, at the optimal utilization rate, the total "depreciation" of the stock of inventories is now larger, since usage "subtracts" from the stock of inventories that can be carried into next period.

The optimality condition for accumulation of M_t (previously equation 8 in Appendix A) becomes now:

$$\lambda_t\left(1 + \frac{\psi_M}{\delta_M + z_M}\frac{\Delta M_t}{M_{t-1}}\right) = \beta\lambda_{t+1}\left(1 + \frac{\psi_M}{2\left(\delta_M + z_M\right)}\frac{\Delta M_{+1}^2}{M_t^2} - \delta_{Mt} + \frac{\theta_g\left(1-\sigma\right)Y_{gt+1}H_{t+1}^{\nu}}{\varepsilon_{Mt+1}^{\nu}z_{Mt}^{\nu}M_t^{1+\nu}}\right) \tag{41}$$

where $\Delta M_t = M_t - M_{t-1}$. The equation for the optimal usage of inventories (which implicitly pins down the optimal value of z_{Mt}) satisfies:

$$\theta_g\frac{(1-\sigma)\,Y_{gt}H_t^{\nu}}{z_{Mt}^{1+\nu}M_{t-1}^{\nu}} = \left(1 + R_M\left(\frac{1-\zeta_M}{z_M} + \frac{\zeta_M}{z_M^2}z_{Mt}\right)\right)M_{t-1}. \tag{42}$$

Relative to the baseline model spelled out in Appendix A, this new model features one additional equation (equation 42 above) and one additional endogenous variable (z_{Mt}).

[6] The assumption of convexity has two appealing properties: first, it allows us to solve the model using standard perturbation methods; second, and most importantly, it captures the idea that, at the margin, a higher utilization rate leads to a higher depreciation. Note that there are some analogies with the way we write down the utilization function for fixed capital. For fixed capital, we assume that the optimal (steady state) utilization rate of capital is unity, and normalize the utilization function so that no resources are wasted at the optimal utilization rate. Instead, here we normalize the function a_{Mt} so that the optimal steady state utilization rate is less than unity.