Appendix C: Estimation Details for "Housing Market Spillovers: Evidence from an Estimated DSGE Model"

Matteo Iacoviello[*] Stefano Neri[†]
Boston College Bank of Italy

March 3, 2009

[*]iacoviel@bc.edu. Address: Boston College, Department of Economics, 140 Commonwealth Ave, Chestnut Hill, MA, 02467, USA.

[†]stefano.neri@bancaditalia.it. Address: Banca d'Italia, Research Department, Via Nazionale 91, 00184 Roma, Italy.

Appendix C: Estimation Details for "Housing Market Spillovers: Evidence from an Estimated DSGE Model"

1 Estimation Strategy

The parameters of the model are estimated using Bayesian methods. We use Bayesian methods because they allow incorporating a priori information on the parameters of the model and also because pure maximum likelihood tends to produce fragile results, particularly in situations in which some parameters are weakly identified.

2 Estimation of the model

Before estimating the model a transformation of the data that is consistent with the balanced-growth path assumption must be taken. Let a sans-serif denote the detrended variables, that is the variables scaled by their deterministic trend. Therefore: $\mathsf{C}_t = C_t/G_C^t$, $\mathsf{IH}_t = IH_t/G_{IH}^t$, $\mathsf{IK}_t = IK_t/G_{IK}^t$, $\mathsf{q}_t = q_t/G_q^t$. Let a superscript d denote the data (see Appendix A for data sources). The measurement equations are:

$$
\begin{aligned}
\log C_t^d - \log C_{1965:1}^d &= \widehat{\mathsf{C}}_t + (G_C - 1)\, t \\
\log IK_t^d - \log IK_{1965:1}^d &= \widehat{\mathsf{IK}}_t + (G_{KC} - 1)\, t \\
\log IH_t^d - \log IH_{1965:1}^d &= \widehat{\mathsf{IH}}_t + (G_H - 1)\, t \\
\log q_t^d - \log q_{1965:1}^d &= \widehat{\mathsf{q}}_t + (G_Q - 1)\, t \\
\log N_{ct}^d &= \alpha \widehat{n}_{ct} + (1-\alpha)\, \widehat{n}'_{ct} \\
\log N_{ht}^d &= \alpha \widehat{n}_{ht} + (1-\alpha)\, \widehat{n}'_{ht} \\
\pi_t^d &= \widehat{\pi}_t \\
R_t^d &= \widehat{R}_t \\
\omega_{ct}^d &= \frac{w_c}{w_c + w'_c} \widehat{\omega}_{ct} + \frac{w'_c}{w_c + w'_c} \widehat{\omega}'_{ct} \\
\omega_{ht}^d &= \frac{w_h}{w_h + w'_h} \widehat{\omega}_{ht} + \frac{w'_h}{w_h + w'_h} \widehat{\omega}'_{ht}
\end{aligned}
$$

where real consumption, real business fixed investment and real residential investment are divided by the civilian non-institutional population over 16 (CNP16OV), and a hat over a variable denotes its percentage deviation from the steady state, detrended value. The first observation is taken away from the trending series since we do not use information on the long-run averages of the detrended data.

3 The simulation of the posterior with the Metropolis algorithm

In the Bayesian framework both the data Y and of the parameters Θ are random variables. Starting from their joint probability distribution $P(Y, \Theta)$ one can derive the relationship between their marginal and conditional distributions, i.e. the Bayes theorem:

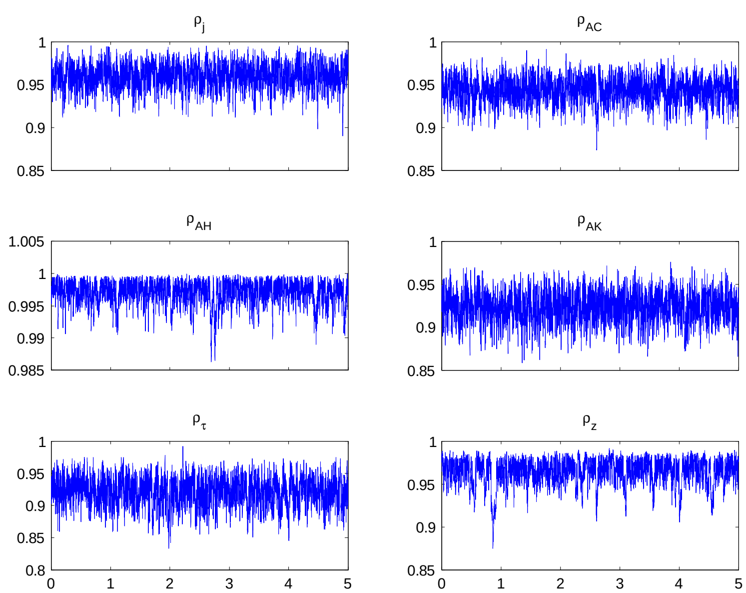
$$P(\Theta|Y) \propto P(Y|\Theta) * P(\Theta)$$

The information contained in the prior distribution $P(\Theta)$ is updated with the likelihood, $P(Y|\Theta)$, of the observed data to deliver the posterior distribution of the parameters $P(\Theta|Y)$. The posterior density can then be used to perform statistical inference either on the parameters themselves or on any function of them.

However, since the posterior distribution of the parameters does not belong to any known family of distributions we need to build our inference on a (Monte Carlo) simulation algorithm that generates a vector of draws from an unknown distribution using a known distribution. As the length of the simulation increases the Markow chain produced by the algorithm converges to the true unknown "target" distribution. The most commonly used algorithm for this purpose is the Metropolis one.[1] As in Schorfheide (2000)[2] and Smets and Wouters (2007), inference is done in two steps. First we maximize the log of the posterior density and compute an approximation of the inverse of the Hessian at the mode. Second, we generate 200,000 draws from the posterior distribution of the parameters using a multivariate normal (the so-called "jump" distibution) with covariance matrix proportional to the inverse of the Hessian. The constant of proportionality is called "scaling" factor. This factor is set at 0.2 and it results in an acceptance rate of 27 percent in 200,000 draws. The first 50,000 draws are used as burn-in sample. The inference described in the paper is based on a total of 150,000 draws from the posterior distribution.

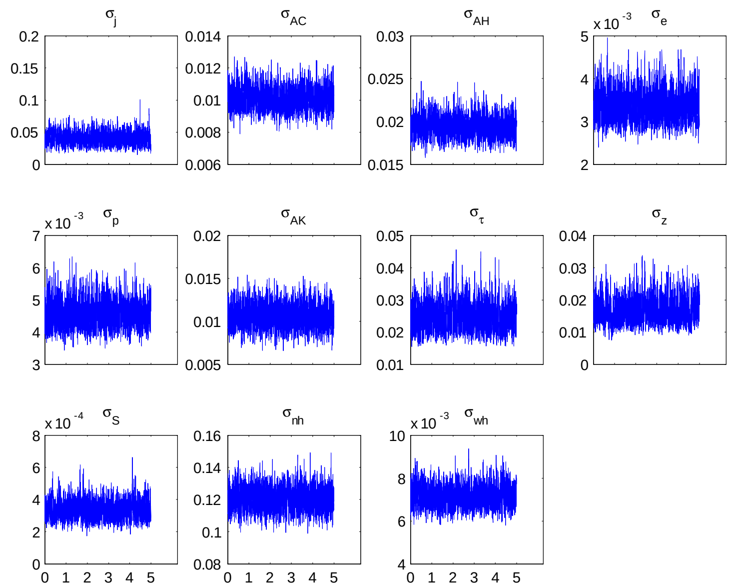
3.1 The output of the Metropolis

The following graphs report the time series of the draws from the posterior distribution generated by the Metropolis algorithm. On the horizontal axis each tick denotes 1,600 draws.

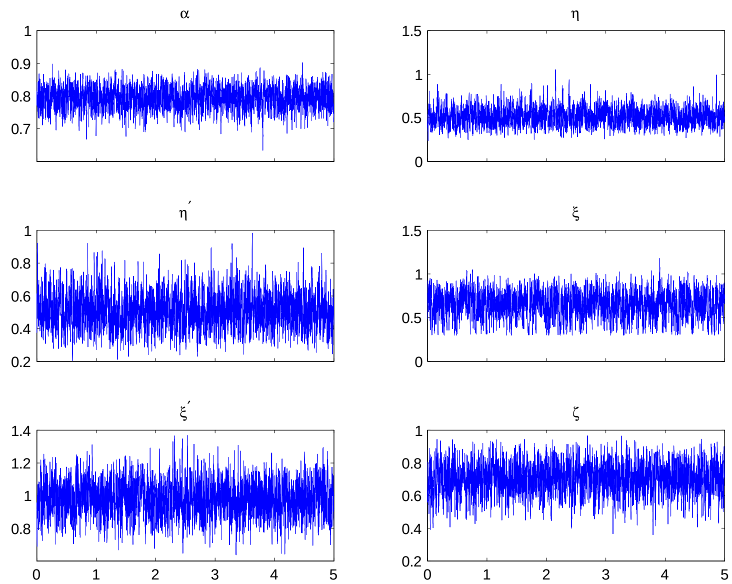


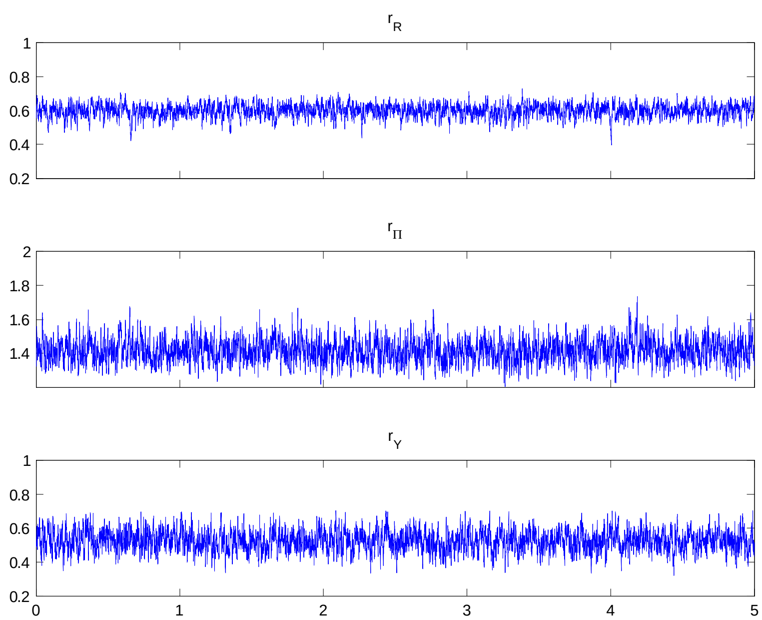
[1] N. Metropolis, A.W. Rosenbluth, M.N. Rosenbluth, A.H. Teller, and E. Teller, "Equations of State Calculations by Fast Computing Machines", *Journal of Chemical Physics*, 21(6):1087-1092, 1953.

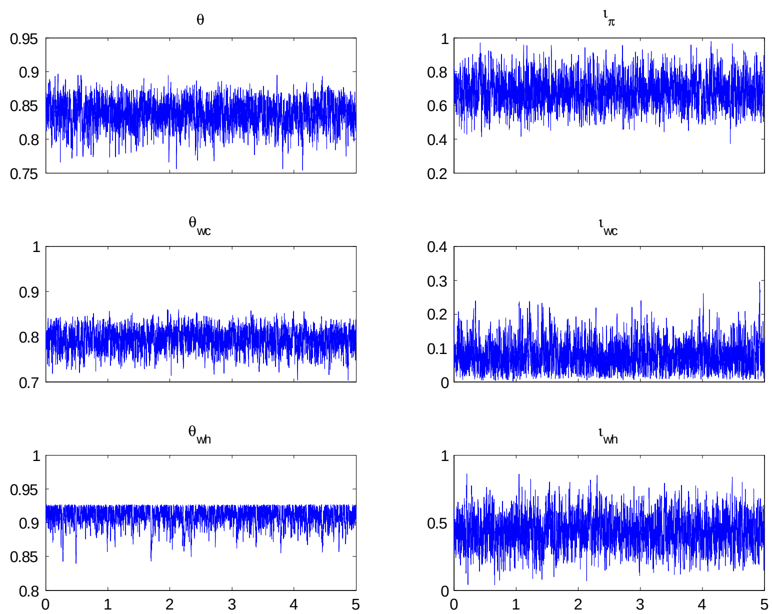
[2] Frank Schorfheide, 2000. "Loss function-based evaluation of DSGE models," Journal of Applied Econometrics, vol. 15(6), pages 645-670.

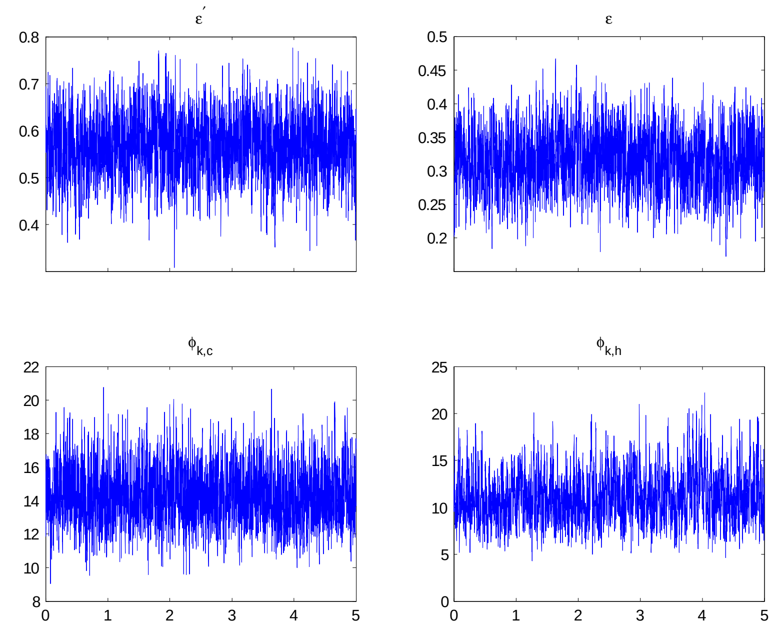


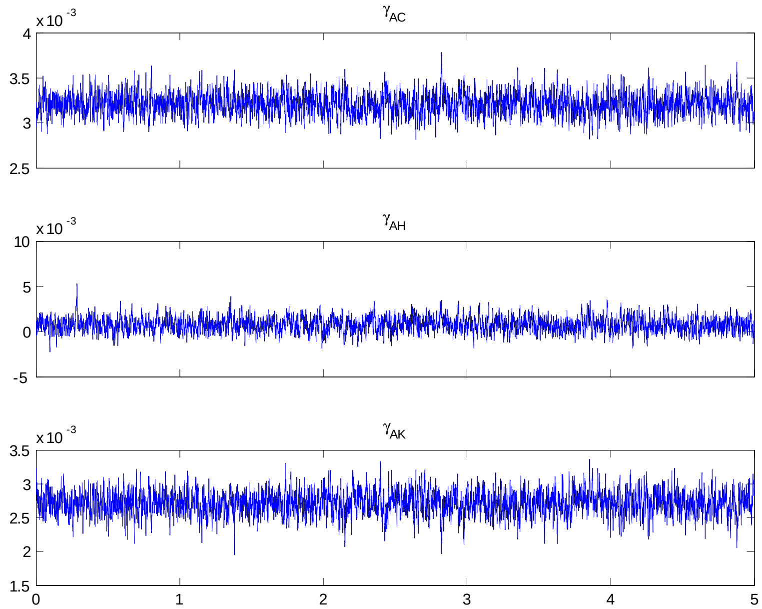
σj
σAC
σAH
σe
σp
σAK
στ
σz
σS
σnh
σwh


α
η
η´
ξ
ξ´
ζ
0
1
2
3
4
5


r_R
r_Π
r_Y
0
1
2
3
4
5


θ
ι_π
θ_wc
ι_wc
θ_wh
ι_wh


ε′
ε
ϕ_k,c
ϕ_k,h



γAC
x 10^-3
4
3.5
3
2.5
γAH
x 10^-3
10
5
0
-5
γAK
x 10^-3
3.5
3
2.5
2
1.5
0
1
2
3
4
5

3.2 Convergence of the algorithm

Convergence of the algorithm is assessed by looking at the plots of the draws, the first four moments (mean, standard deviation, skewness and kurtosis) obtained by splitting the draws into two samples (first and second half) and by computing recursively the first four moments of the marginal posterior distribution of each parameter. Table C.1 reports the first and second moments of the posterior marginal distributions based on 200,000 draws. Table C.2 reports the moments based on 500,000 draws.

Table C.1. Posterior mean and standard deviation: 200,000 draws

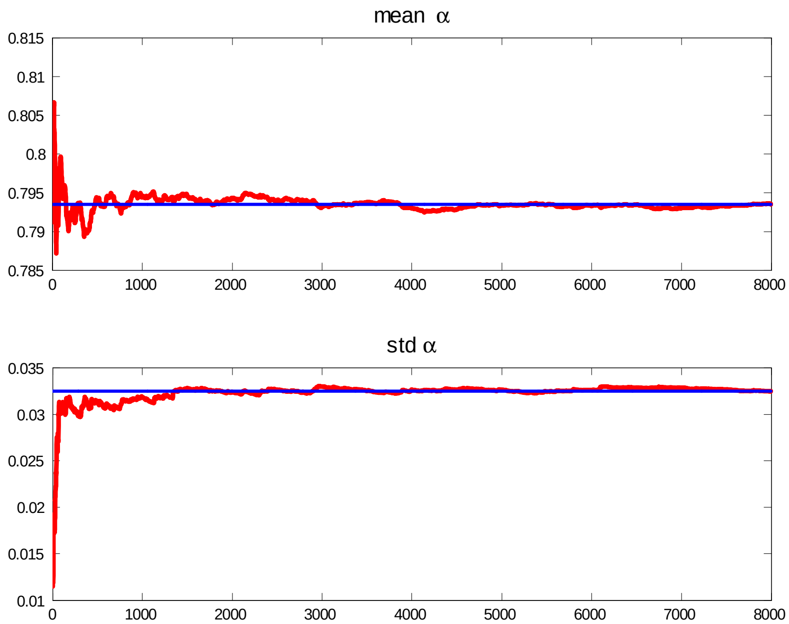
	mean		standard deviation	
parameter	first half	second half	first half	second half
ϵ	0.3170	0.3142	0.0409	0.0396
ϵ'	0.5659	0.5680	0.0620	0.0607
η	0.5169	0.5179	0.1025	0.0928
η'	0.5106	0.5112	0.1020	0.1021
ξ	0.6521	0.6474	0.1434	0.1443
ξ'	0.9821	0.9776	0.1018	0.0996
$\phi_{k,c}$	14.3652	14.2922	1.6103	1.5578
$\phi_{k,h}$	10.9398	11.4178	2.4066	2.6513
α	0.7930	0.7940	0.0326	0.0323
r_R	0.5996	0.5999	0.0393	0.0376
r_π	1.4175	1.4161	0.0661	0.0673
r_Y	0.5281	0.5222	0.0602	0.0585
θ_π	0.8381	0.8363	0.0202	0.0193
ι_π	0.6814	0.6847	0.0868	0.0860
$\theta_{w,c}$	0.7951	0.7927	0.0238	0.0234
$\iota_{w,c}$	0.0796	0.0793	0.0414	0.0400
$\theta_{w,h}$	0.9091	0.9091	0.0139	0.0126
$\iota_{w,h}$	0.4360	0.4348	0.1210	0.1173
ζ	0.7021	0.7039	0.0915	0.0884
γ_{AC}	0.0032	0.0032	0.0001	0.0001
γ_{AH}	0.0008	0.0008	0.0008	0.0008
γ_{AK}	0.0027	0.0027	0.0002	0.0002
ρ_{AC}	0.9436	0.9427	0.0141	0.0144
ρ_{AH}	0.9970	0.9968	0.0017	0.0020
ρ_{AK}	0.9224	0.9231	0.0178	0.0164
ρ_j	0.9594	0.9599	0.0138	0.0144
ρ_z	0.9624	0.9648	0.0163	0.0149
ρ_τ	0.9213	0.9204	0.0218	0.0216
σ_{AC}	0.0102	0.0101	0.0007	0.0006
σ_{AH}	0.0195	0.0194	0.0011	0.0011
σ_{AK}	0.0106	0.0105	0.0013	0.0012
σ_j	0.0408	0.0404	0.0097	0.0100
σ_R	0.0034	0.0034	0.0003	0.0003
σ_z	0.0168	0.0173	0.0041	0.0040
σ_τ	0.0255	0.0252	0.0045	0.0043
σ_p	0.0046	0.0046	0.0004	0.0004
σ_s	0.0003	0.0003	0.0001	0.0001
$\sigma_{n,h}$	0.1207	0.1206	0.0068	0.0068
$\sigma_{w,h}$	0.0072	0.0072	0.0005	0.0005

Table C.2. Posterior mean and standard deviation: 500,000 draws

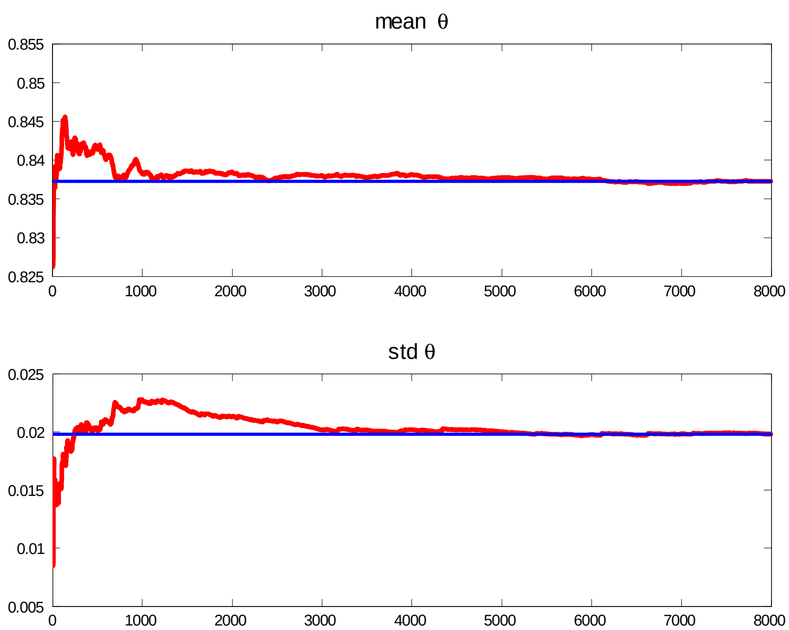
	mean		standard deviation	
parameter	first half	second half	first half	second half
ϵ	0.3198	0.3181	0.0410	0.0407
ϵ'	0.5672	0.5676	0.0620	0.0630
η	0.5149	0.5155	0.0980	0.1000
η'	0.5105	0.5138	0.1020	0.1015
ξ	0.6453	0.6403	0.1416	0.1385
ξ'	0.9773	0.9823	0.1022	0.1012
$\phi_{k,c}$	14.3400	14.2892	1.5737	1.5672
$\phi_{k,h}$	11.1922	11.1316	2.5818	2.5239
α	0.7929	0.7928	0.0321	0.0329
r_R	0.6012	0.5993	0.0389	0.0377
r_π	1.4173	1.4192	0.0669	0.0676
r_Y	0.6012	0.5993	0.0608	0.0622
θ_π	0.8371	0.8379	0.0193	0.0188
ι_π	0.6845	0.6779	0.0844	0.0859
$\theta_{w,c}$	0.7954	0.7939	0.0254	0.0256
$\iota_{w,c}$	0.0802	0.0831	0.0400	0.0417
$\theta_{w,h}$	0.9092	0.9091	0.0128	0.0125
$\iota_{w,h}$	0.4355	0.4312	0.1204	0.1224
ζ	0.7007	0.6941	0.0901	0.0960
γ_{AC}	0.0032	0.0032	0.0001	0.0001
γ_{AH}	0.0008	0.0008	0.0008	0.0008
γ_{AK}	0.0027	0.0027	0.0002	0.0002
ρ_{AC}	0.9423	0.9427	0.0147	0.0144
ρ_{AH}	0.9968	0.9969	0.0020	0.0021
ρ_{AK}	0.9229	0.9230	0.0173	0.0169
ρ_j	0.9597	0.9601	0.0143	0.0137
ρ_z	0.9629	0.9643	0.0176	0.0151
ρ_τ	0.9185	0.9208	0.0229	0.0226
σ_{AC}	0.0101	0.0101	0.0006	0.0006
σ_{AH}	0.0194	0.0194	0.0011	0.0011
σ_{AK}	0.0106	0.0105	0.0013	0.0012
σ_j	0.0405	0.0400	0.0098	0.0093
σ_R	0.0034	0.0034	0.0034	0.0034
σ_z	0.0173	0.0174	0.0042	0.0043
σ_τ	0.0259	0.0256	0.0052	0.0049
σ_p	0.0046	0.0046	0.0004	0.0004
σ_s	0.0003	0.0003	0.0001	0.0001
$\sigma_{n,h}$	0.1208	0.1210	0.0068	0.0068
$\sigma_{w,h}$	0.0071	0.0072	0.0005	0.0005

Table C.3. Posterior mean and standard deviation: 200,000 and 500,000 draws

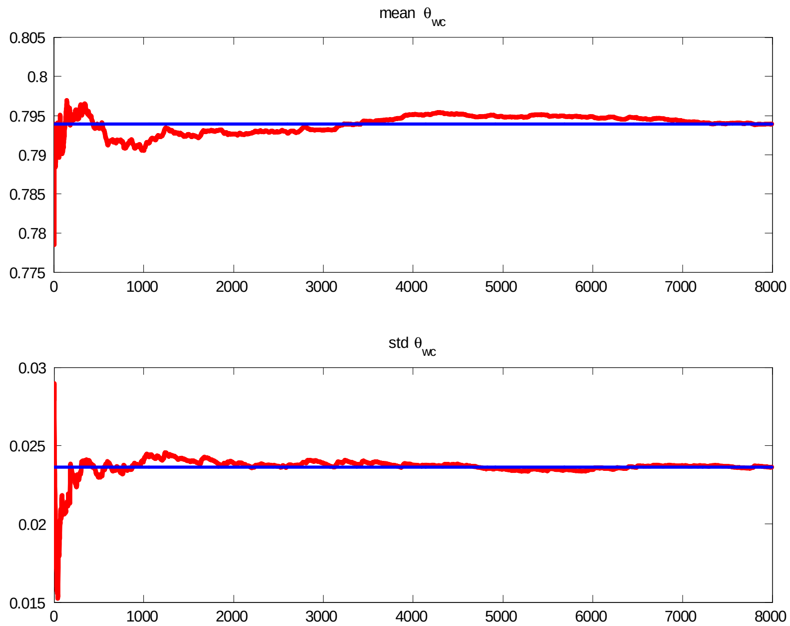
	mean		standard deviation	
	200,000		500,000	
parameter	second half	second half	first half	second half
ϵ	0.3170	0.3198	0.0409	0.0407
ϵ'	0.5659	0.5672	0.0620	0.0630
η	0.5169	0.5149	0.1025	0.1000
η'	0.5106	0.5105	0.1020	0.1015
ξ	0.6521	0.6453	0.1434	0.1385
ξ'	0.9821	0.9773	0.1018	0.1012
$\phi_{k,c}$	14.3652	14.3400	1.6103	1.5672
$\phi_{k,h}$	10.9398	11.1922	2.4066	2.5239
α	0.7930	0.7929	0.0326	0.0329
r_R	0.5996	0.6012	0.0393	0.0377
r_π	1.4175	1.4173	0.0661	0.0676
r_Y	0.5281	0.6012	0.0602	0.0622
θ_π	0.8381	0.8371	0.0202	0.0188
ι_π	0.6814	0.6845	0.0868	0.0859
$\theta_{w,c}$	0.7951	0.7954	0.0238	0.0256
$\iota_{w,c}$	0.0796	0.0802	0.0414	0.0417
$\theta_{w,h}$	0.9091	0.9092	0.0139	0.0125
$\iota_{w,h}$	0.4360	0.4355	0.1210	0.1224
ζ	0.7021	0.7007	0.0915	0.0960
γ_{AC}	0.0032	0.0032	0.0001	0.0001
γ_{AH}	0.0008	0.0008	0.0008	0.0008
γ_{AK}	0.0027	0.0027	0.0002	0.0002
ρ_{AC}	0.9436	0.9423	0.0141	0.0144
ρ_{AH}	0.9970	0.9968	0.0017	0.0021
ρ_{AK}	0.9224	0.9229	0.0178	0.0169
ρ_j	0.9594	0.9597	0.0138	0.0137
ρ_z	0.9624	0.9629	0.0163	0.0151
ρ_τ	0.9213	0.9185	0.0218	0.0226
σ_{AC}	0.0102	0.0101	0.0007	0.0006
σ_{AH}	0.0195	0.0194	0.0011	0.0011
σ_{AK}	0.0106	0.0106	0.0013	0.0012
σ_j	0.0408	0.0405	0.0097	0.0093
σ_R	0.0034	0.0034	0.0003	0.0034
σ_z	0.0168	0.0173	0.0041	0.0043
σ_τ	0.0255	0.0259	0.0045	0.0049
σ_p	0.0046	0.0046	0.0004	0.0004
σ_s	0.0003	0.0003	0.0001	0.0001
$\sigma_{n,h}$	0.1207	0.1208	0.0068	0.0068
$\sigma_{w,h}$	0.0072	0.0071	0.0005	0.0005



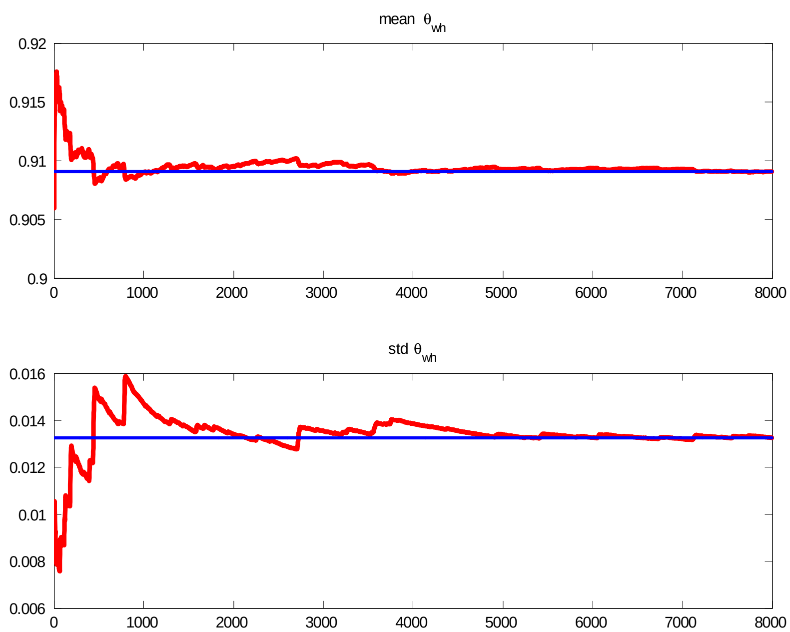
α



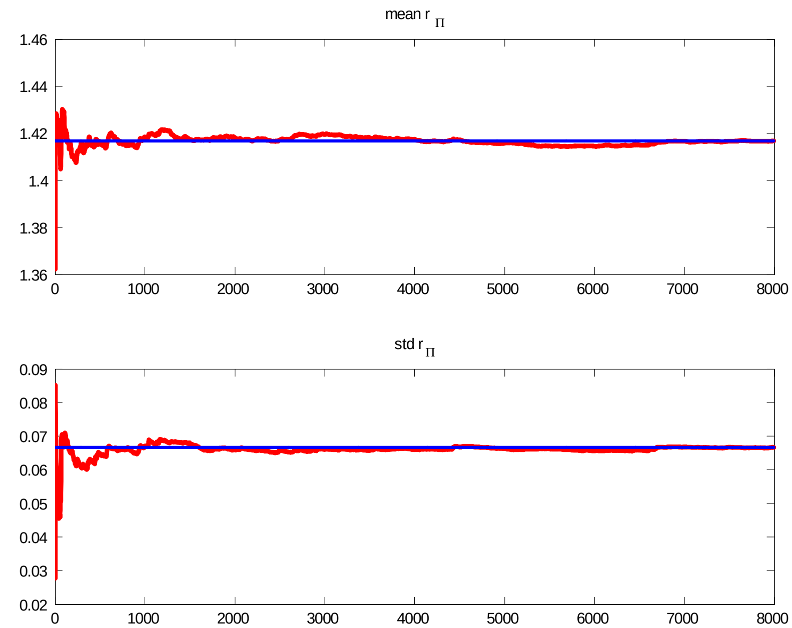
θ_π



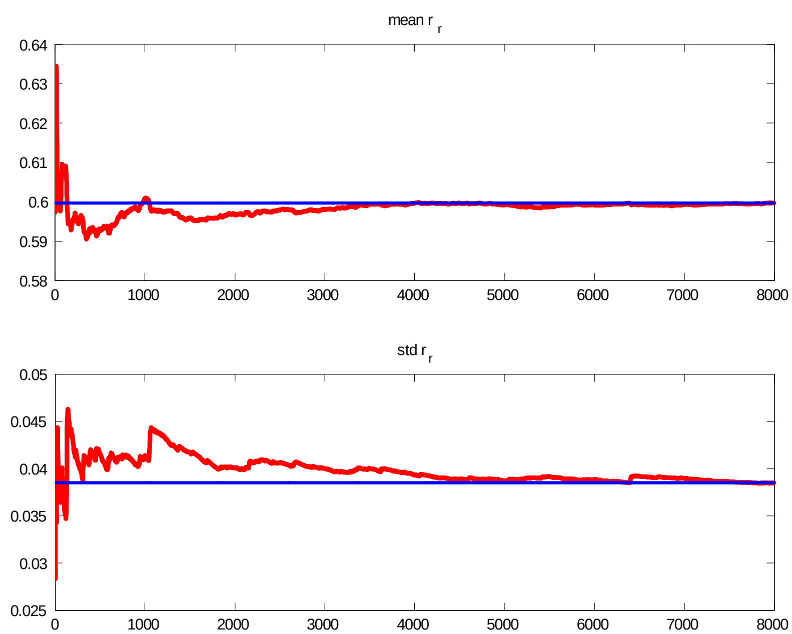
θ_{wc}



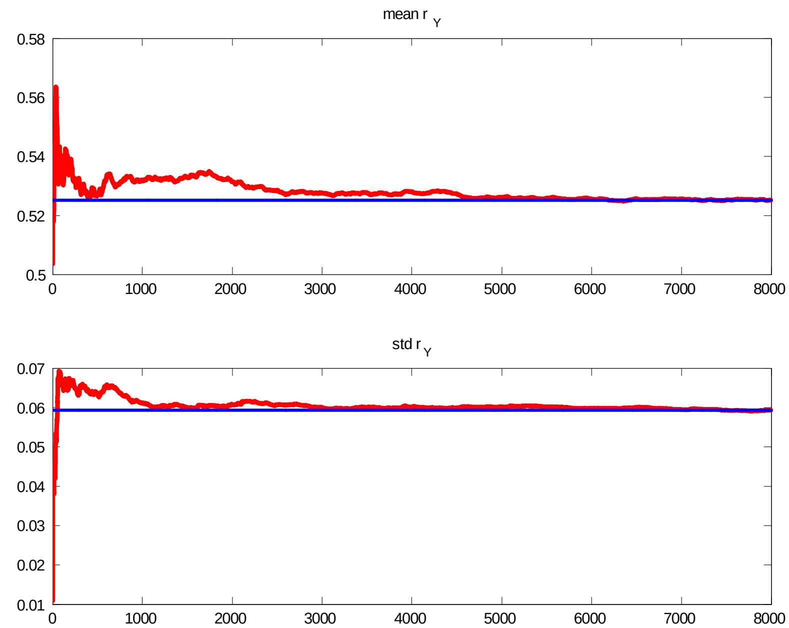
θ_{wh}



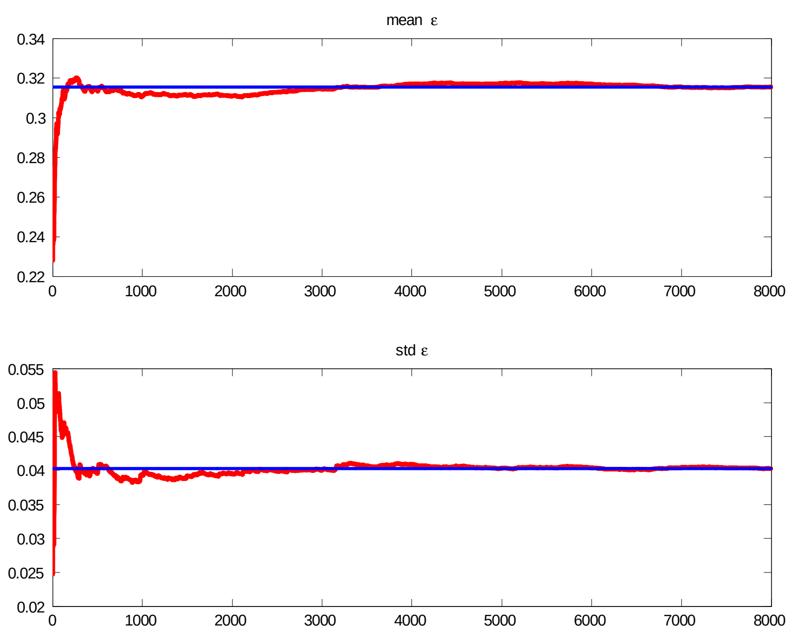
r_π



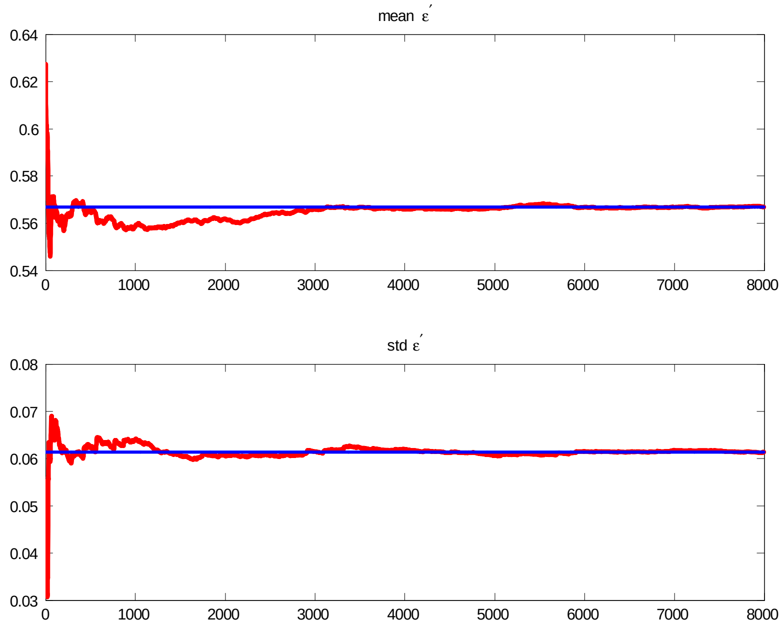
r_R


mean r Y
0.58
0.56
0.54
0.52
0.5
0
1000
2000
3000
4000
5000
6000
7000
8000
std r Y
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1000
2000
3000
4000
5000
6000
7000
8000

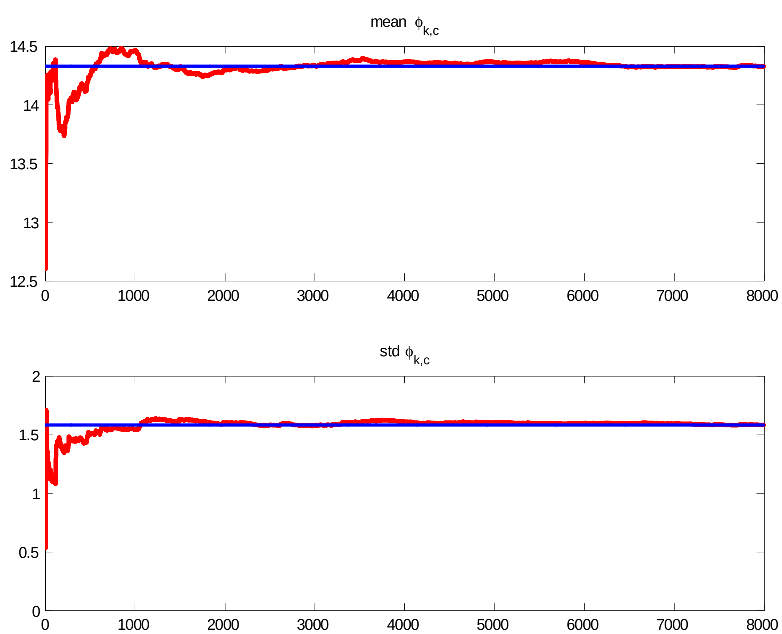
r_Y


mean ε
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0
1000
2000
3000
4000
5000
6000
7000
8000
std ε
0.055
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0
1000
2000
3000
4000
5000
6000
7000
8000

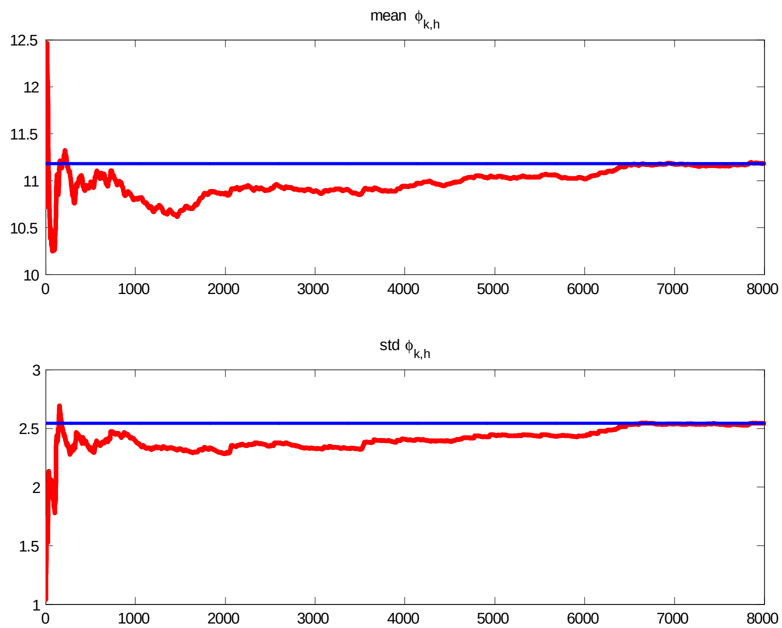
ϵ



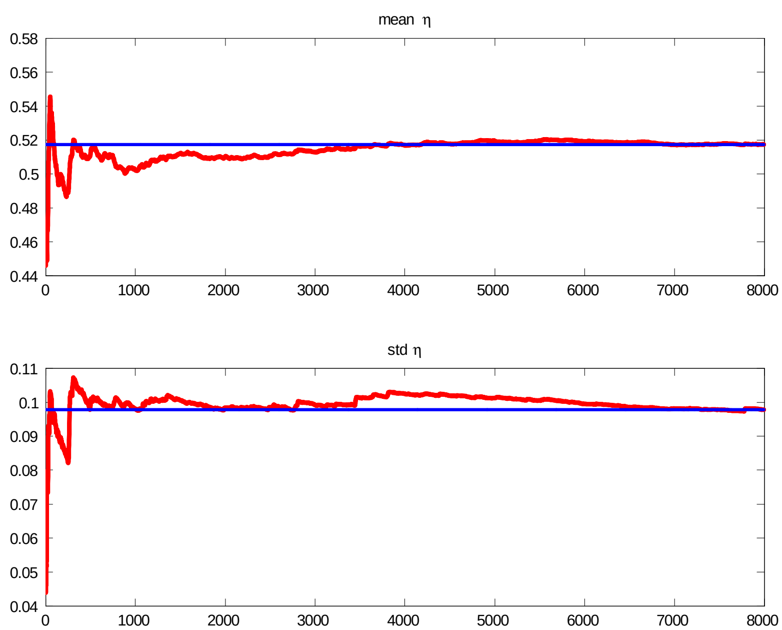
ϵ'



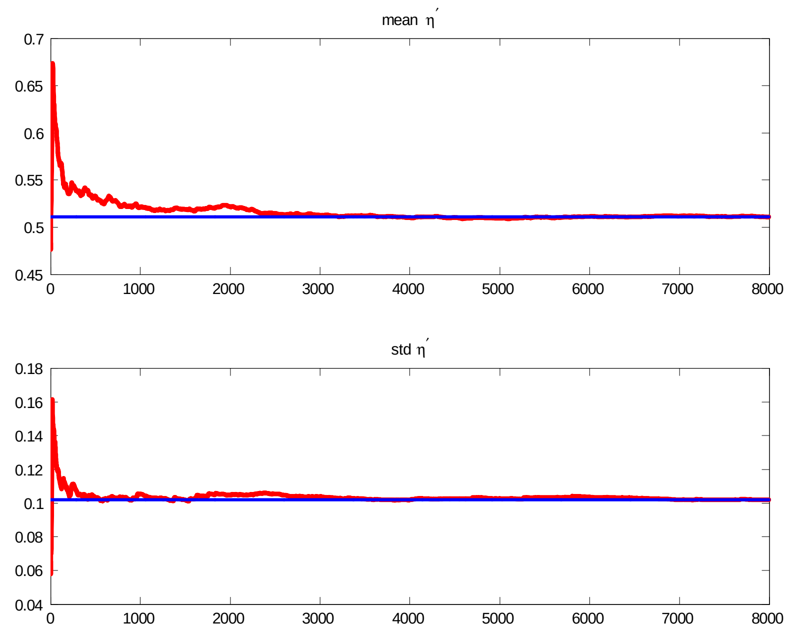
$\phi_{k,c}$


mean φ k,h
12.5
12
11.5
11
10.5
10
0
1000
2000
3000
4000
5000
6000
7000
8000
std φ k,h
3
2.5
2
1.5
1
0
1000
2000
3000
4000
5000
6000
7000
8000

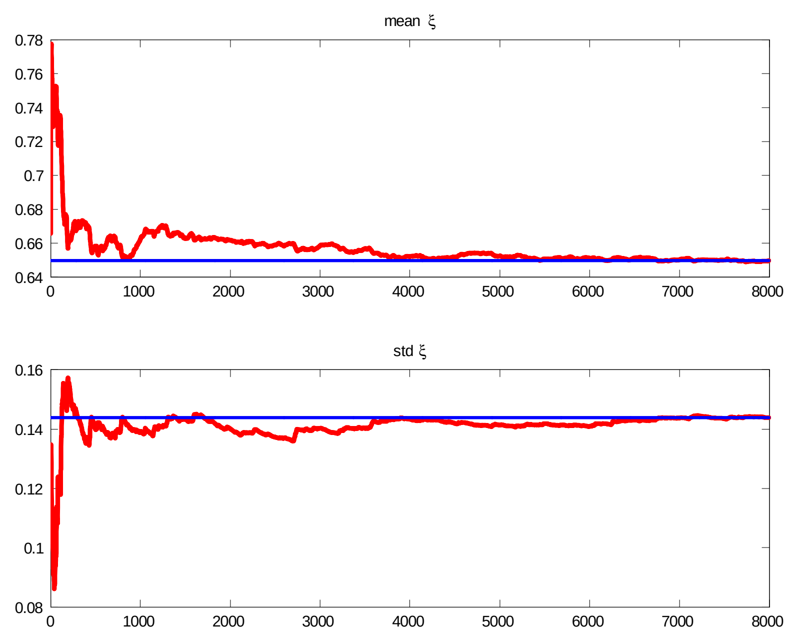
$\phi_{k,h}$


mean η
0.58
0.56
0.54
0.52
0.5
0.48
0.46
0.44
0
1000
2000
3000
4000
5000
6000
7000
8000
std η
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0
1000
2000
3000
4000
5000
6000
7000
8000

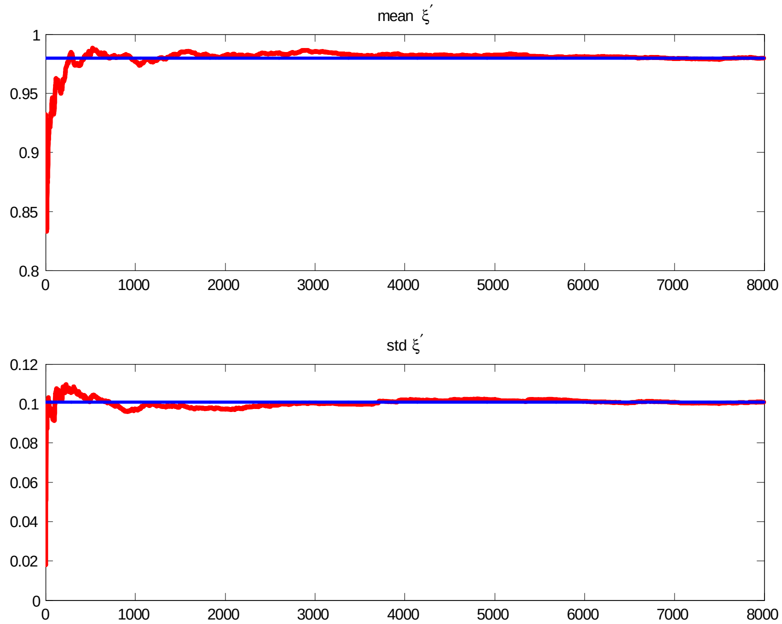
η


mean η´
0.7
0.65
0.6
0.55
0.5
0.45
0
1000
2000
3000
4000
5000
6000
7000
8000
std η´
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0
1000
2000
3000
4000
5000
6000
7000
8000

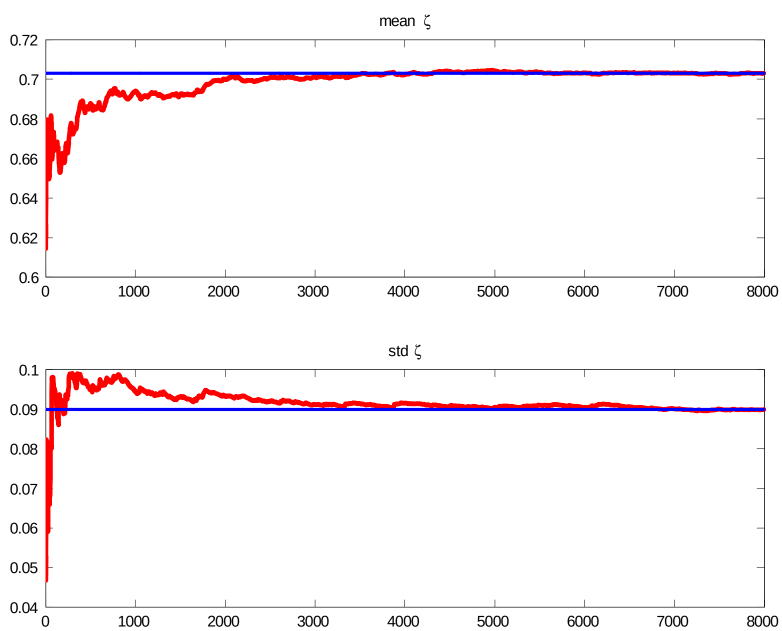
η'


mean ξ
0.78
0.76
0.74
0.72
0.7
0.68
0.66
0.64
0
1000
2000
3000
4000
5000
6000
7000
8000
std ξ
0.16
0.14
0.12
0.1
0.08
0
1000
2000
3000
4000
5000
6000
7000
8000

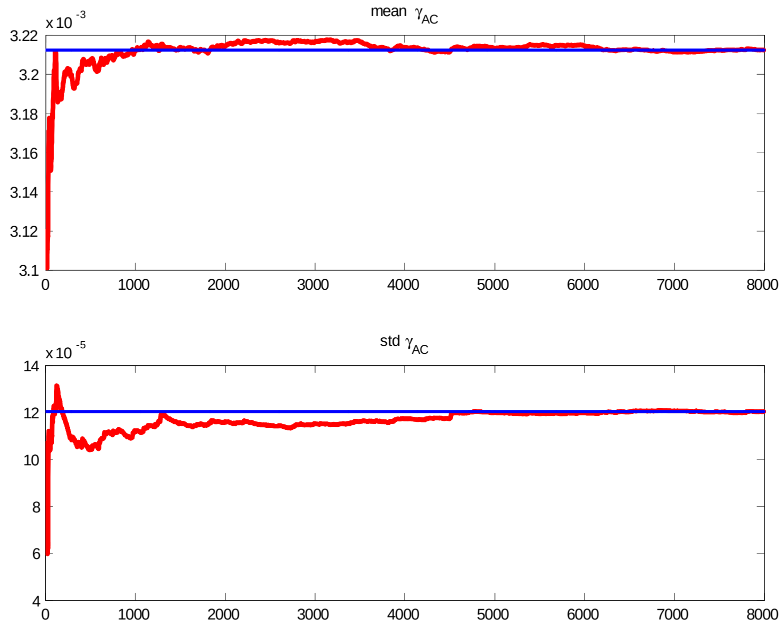
ξ


mean ξ´
1
0.95
0.9
0.85
0.8
0
1000
2000
3000
4000
5000
6000
7000
8000
std ξ´
0.12
0.1
0.08
0.06
0.04
0.02
0
0
1000
2000
3000
4000
5000
6000
7000
8000

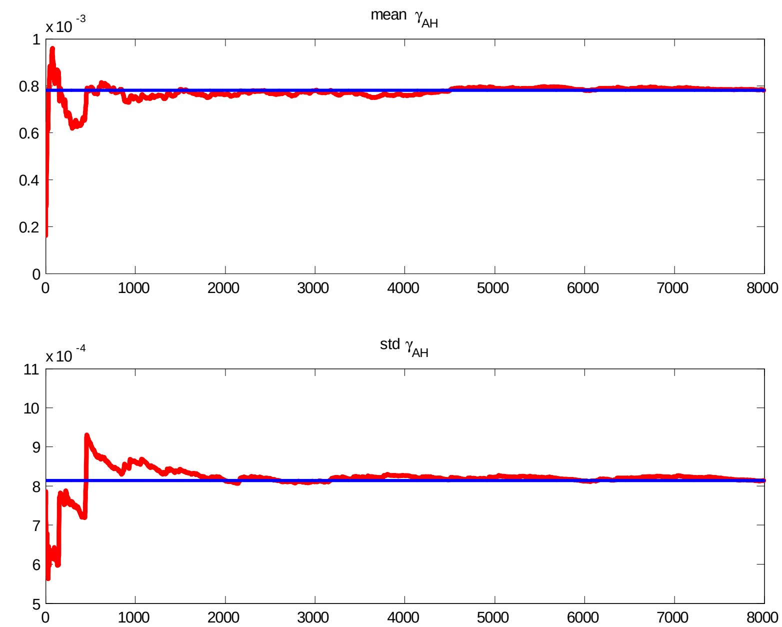
ξ'


mean ζ
0.72
0.7
0.68
0.66
0.64
0.62
0.6
0
1000
2000
3000
4000
5000
6000
7000
8000
std ζ
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0
1000
2000
3000
4000
5000
6000
7000
8000

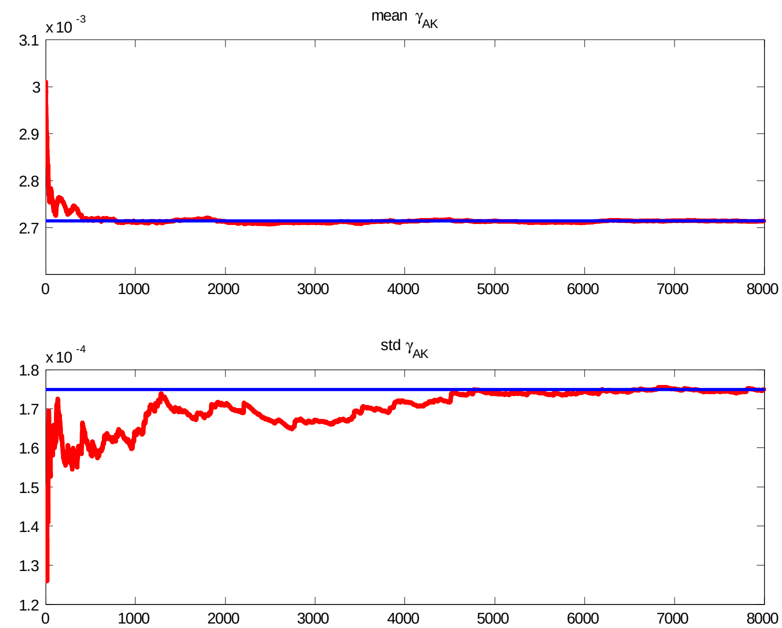
ζ



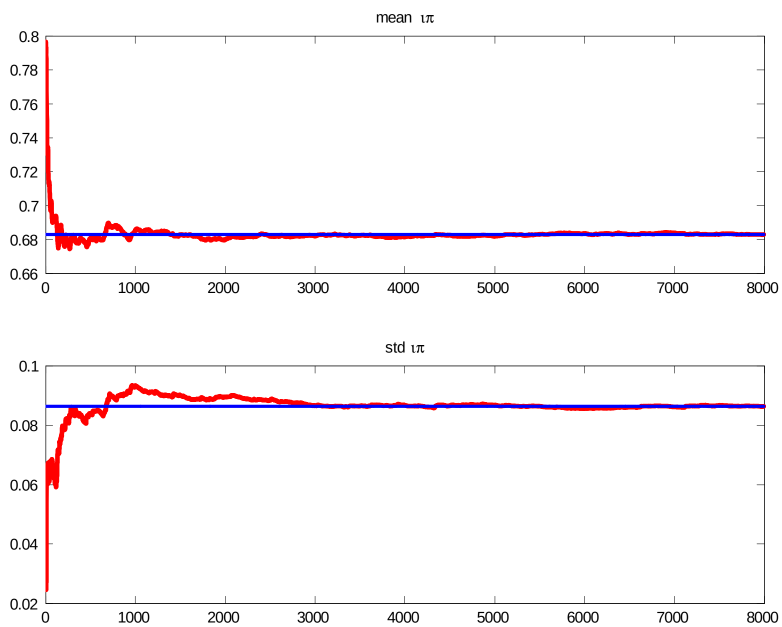
γ_{AC}



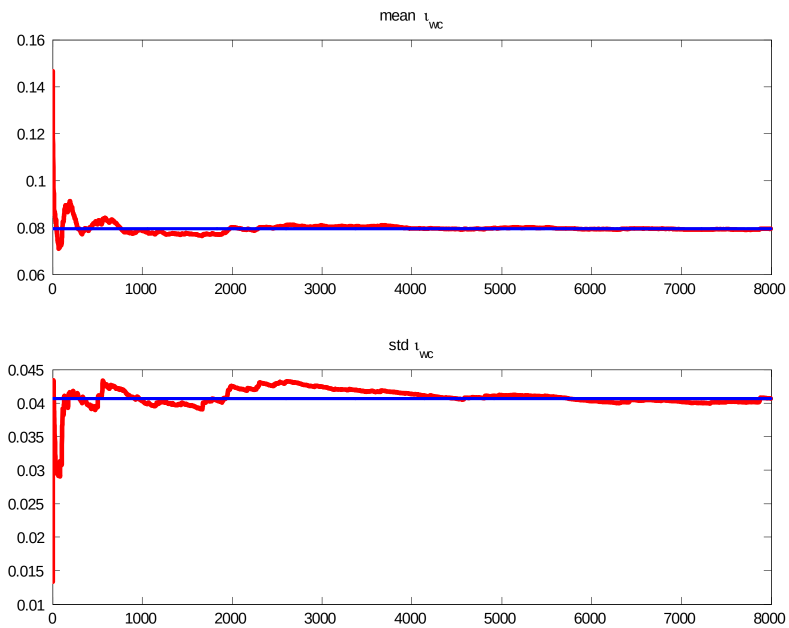
γ_{AH}



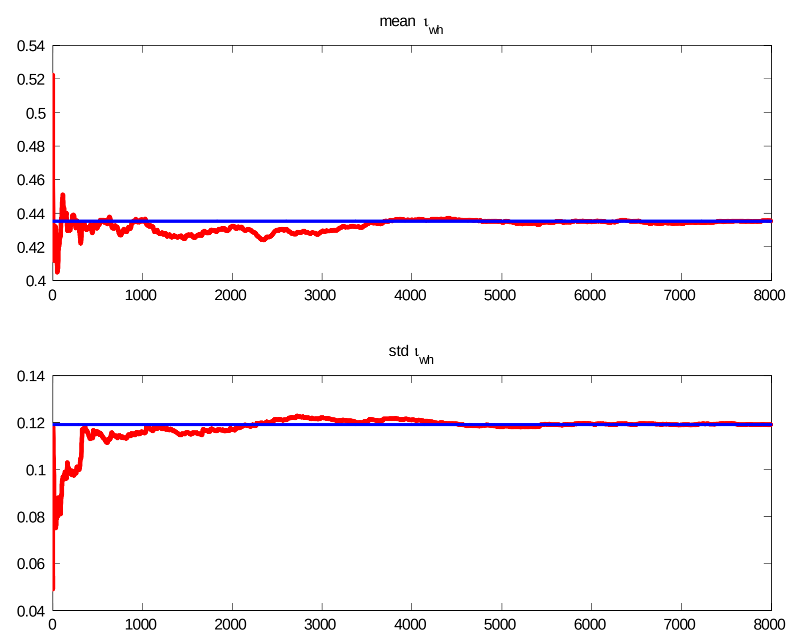
γ_{AK}



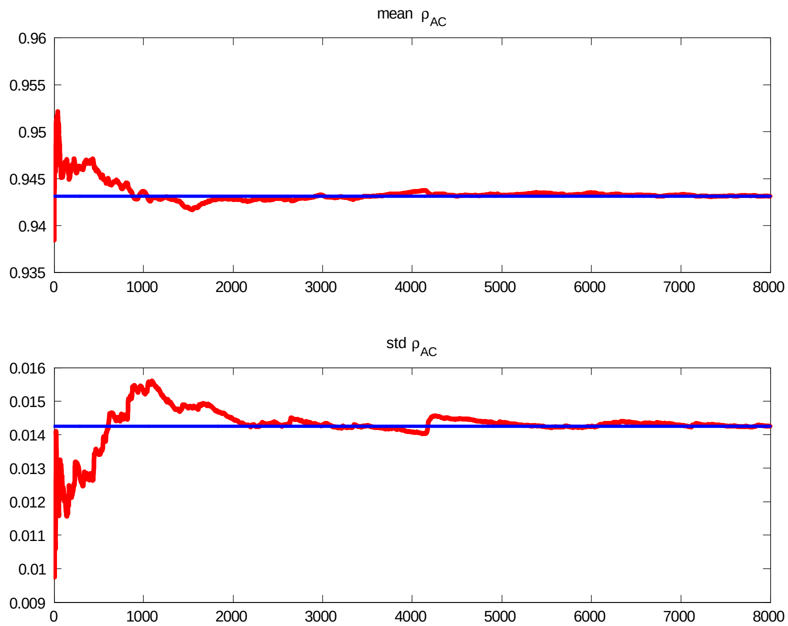
ι_{π}



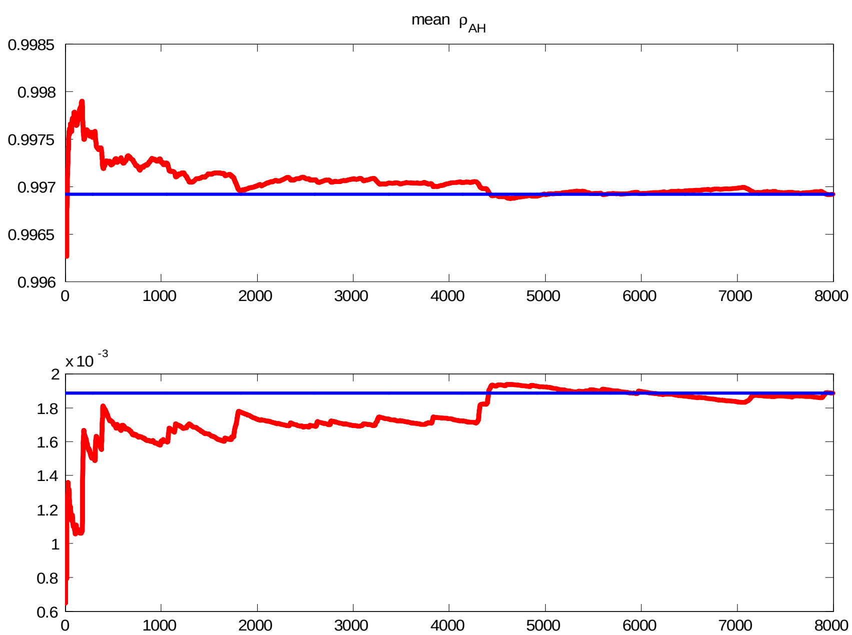
$\iota_{w,c}$



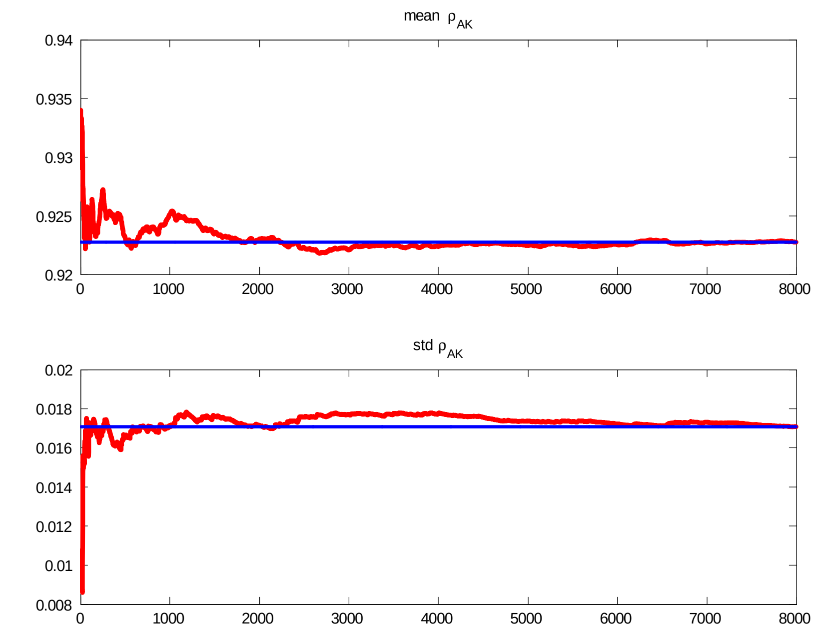
$\iota_{w,h}$


mean ρAC
0.96
0.955
0.95
0.945
0.94
0.935
0 1000 2000 3000 4000 5000 6000 7000 8000
std ρAC
0.016
0.015
0.014
0.013
0.012
0.011
0.01
0.009
0 1000 2000 3000 4000 5000 6000 7000 8000

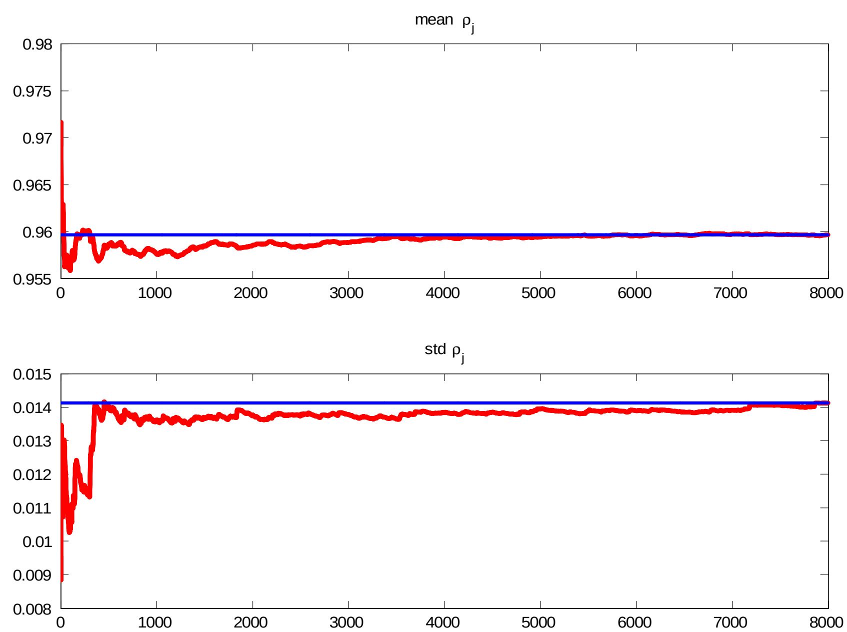
ρ_{AC}


mean ρAH
0.9985
0.998
0.9975
0.997
0.9965
0.996
0 1000 2000 3000 4000 5000 6000 7000 8000
x 10 -3
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0 1000 2000 3000 4000 5000 6000 7000 8000

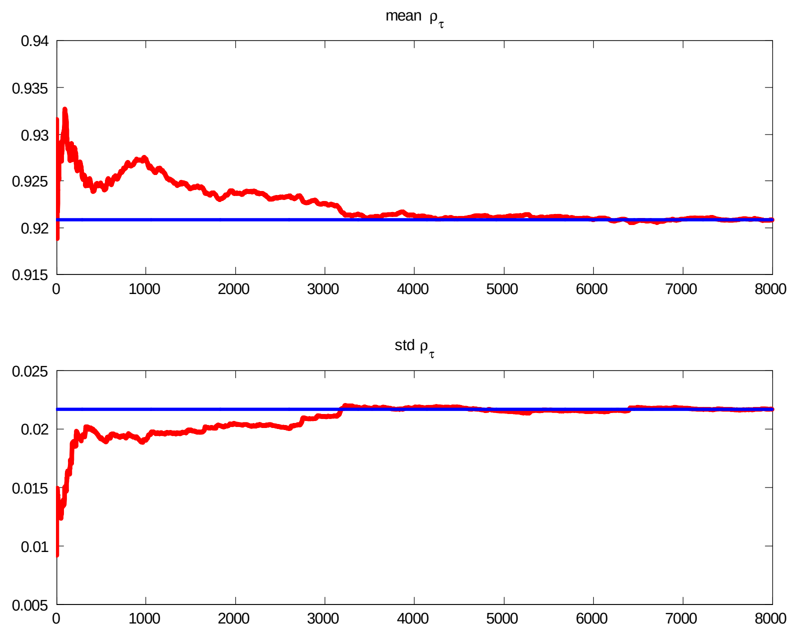
ρ_{AH}


mean ρ AK
0.94
0.935
0.93
0.925
0.92
0
1000
2000
3000
4000
5000
6000
7000
8000
std ρ AK
0.02
0.018
0.016
0.014
0.012
0.01
0.008

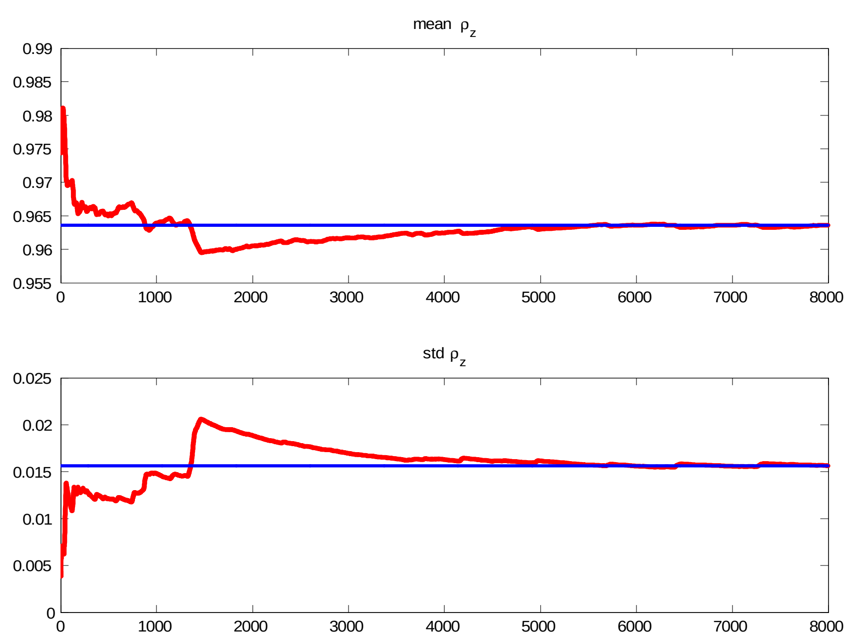
ρ_{AK}


mean ρ j
0.98
0.975
0.97
0.965
0.96
0.955
0
1000
2000
3000
4000
5000
6000
7000
8000
std ρ j
0.015
0.014
0.013
0.012
0.011
0.01
0.009
0.008

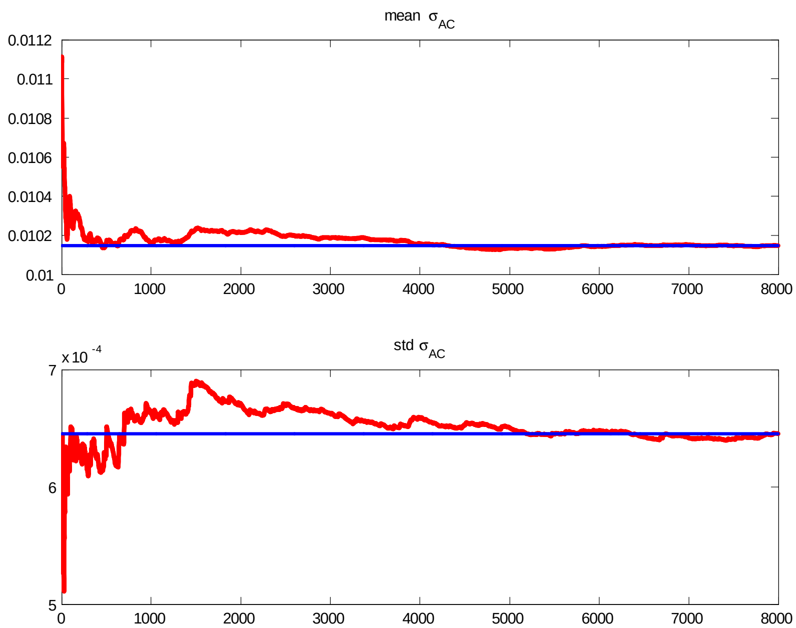
ρ_j


mean ρ_τ
0.94
0.935
0.93
0.925
0.92
0.915
0 1000 2000 3000 4000 5000 6000 7000 8000
std ρ_τ
0.025
0.02
0.015
0.01
0.005
0 1000 2000 3000 4000 5000 6000 7000 8000

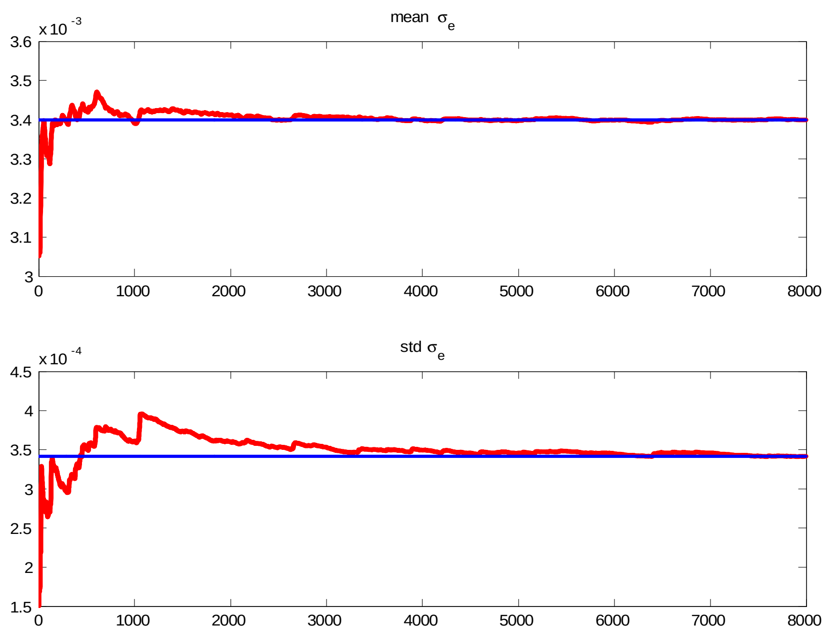
ρ_τ


mean ρ_z
0.99
0.985
0.98
0.975
0.97
0.965
0.96
0.955
0 1000 2000 3000 4000 5000 6000 7000 8000
std ρ_z
0.025
0.02
0.015
0.01
0.005
0
0 1000 2000 3000 4000 5000 6000 7000 8000

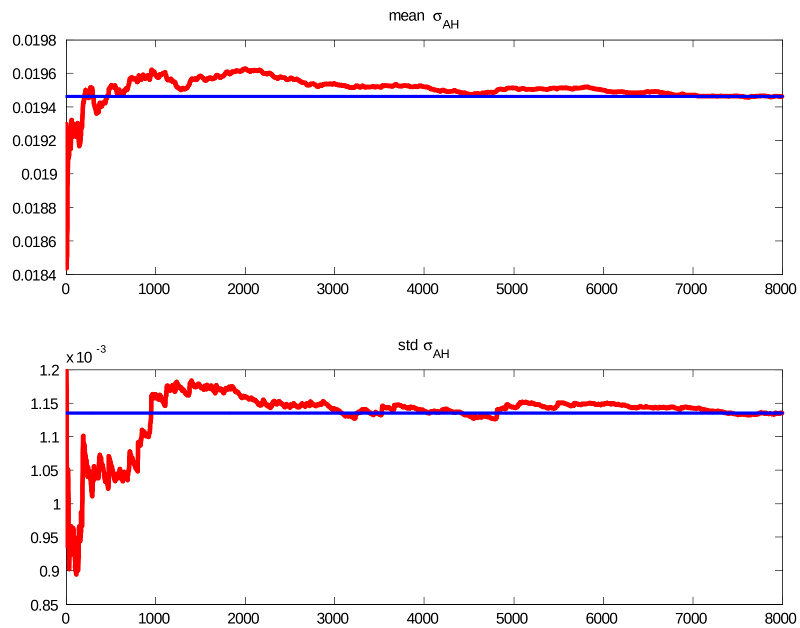
ρ_z



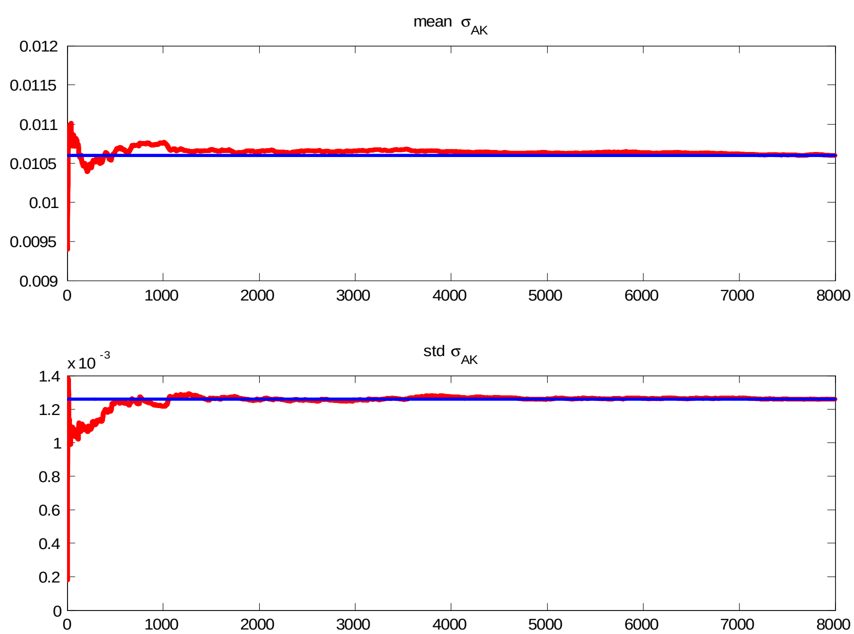
σ_{AC}



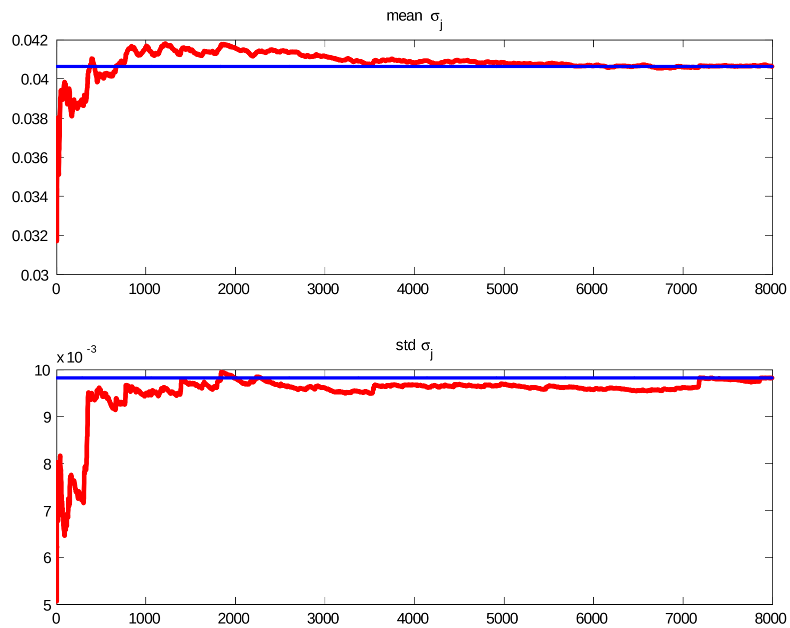
σ_e



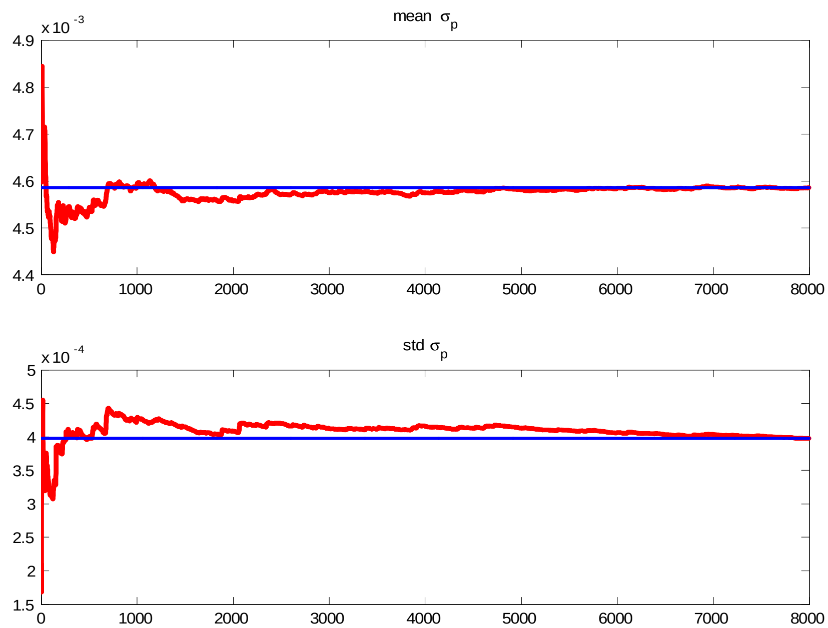
σ_{AH}



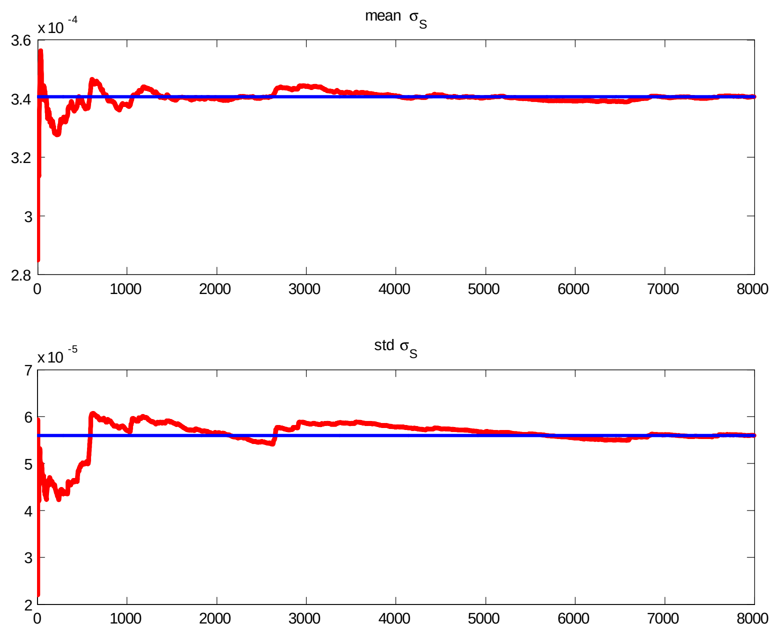
σ_{AK}



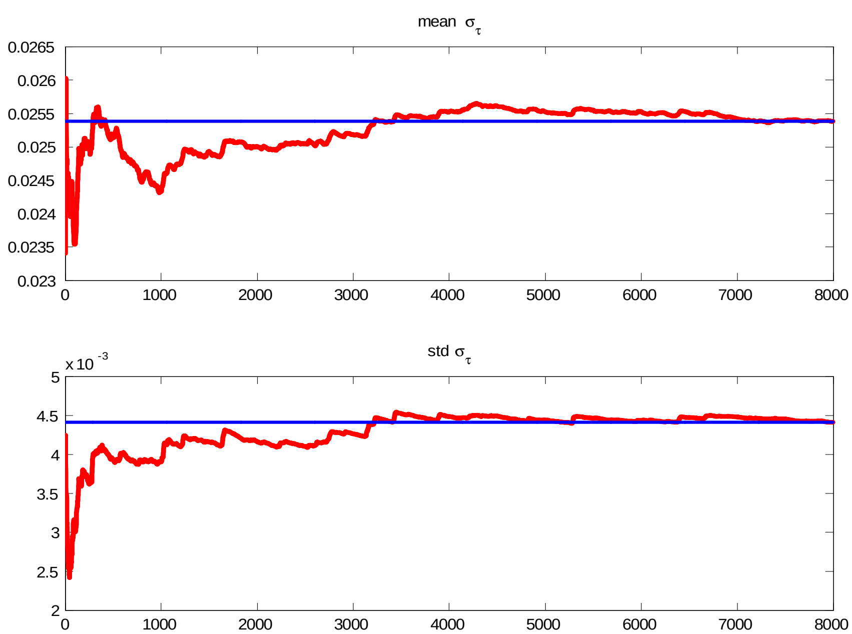
σ_j



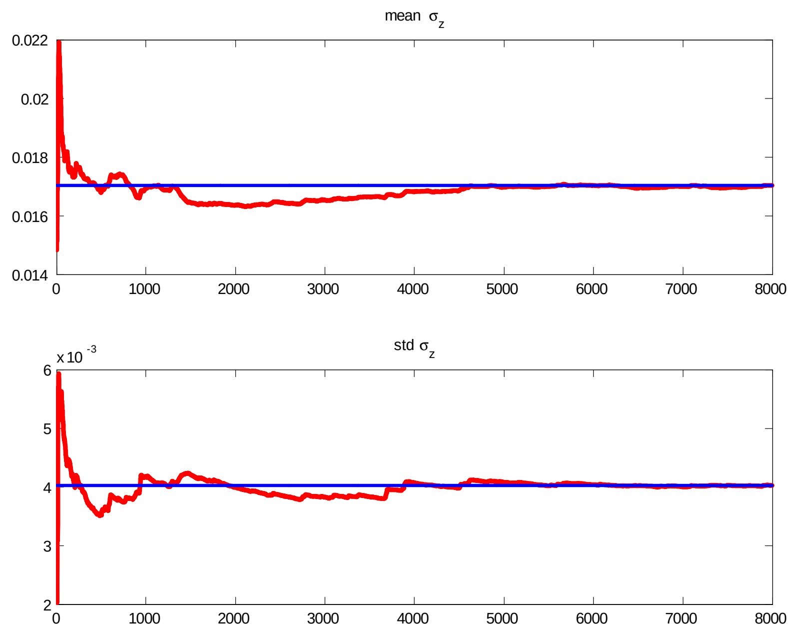
σ_p



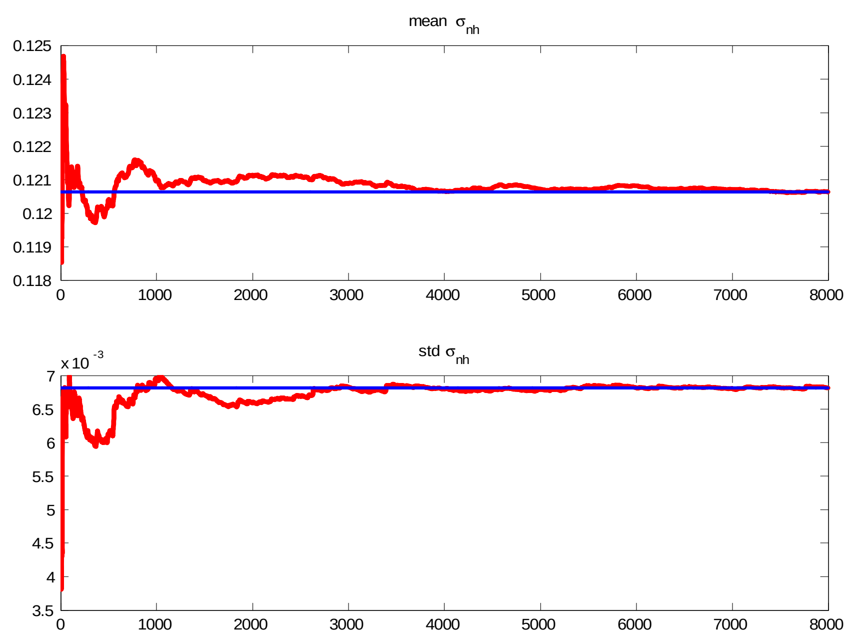
σ_s



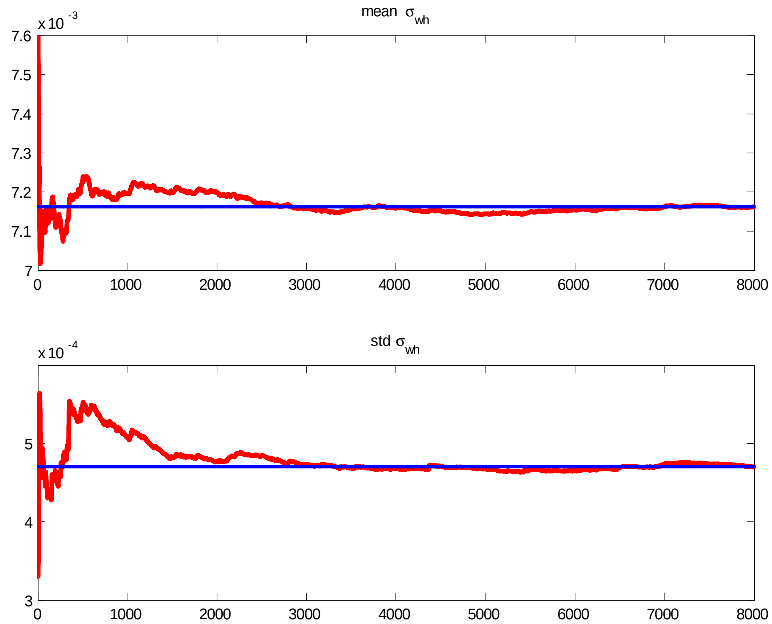
σ_τ



σ_z



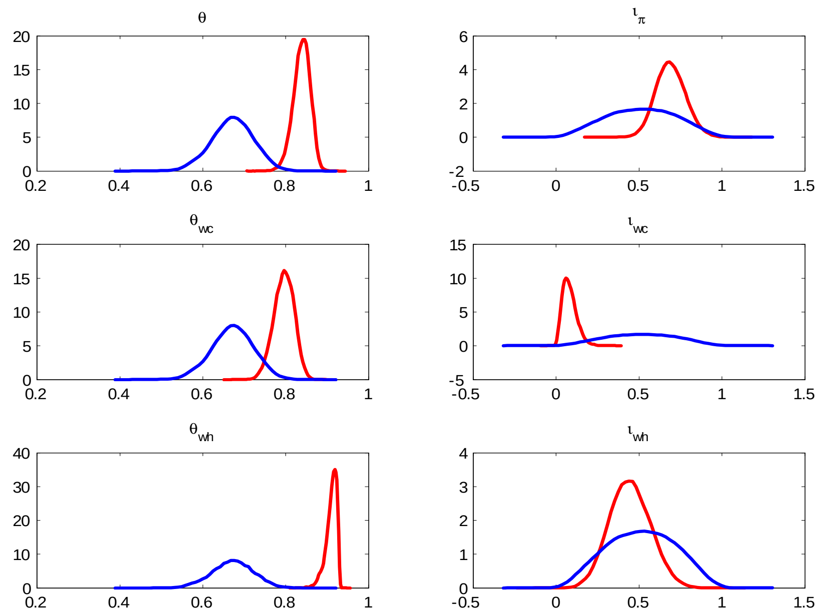
$\sigma_{n,h}$

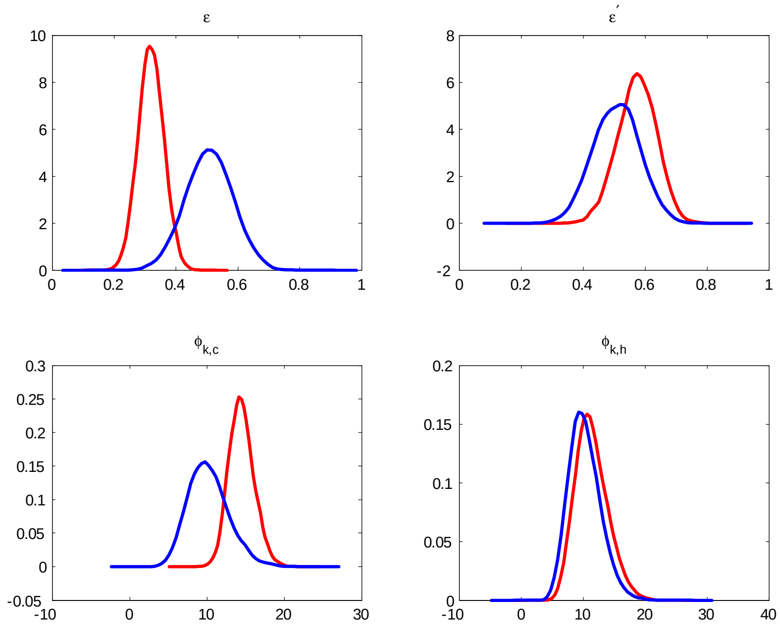

mean σwh
x 10^-3
7.6
7.5
7.4
7.3
7.2
7.1
7
0
1000
2000
3000
4000
5000
6000
7000
8000
std σwh
x 10^-4
5
4
3
0
1000
2000
3000
4000
5000
6000
7000
8000

$\sigma_{n,h}$

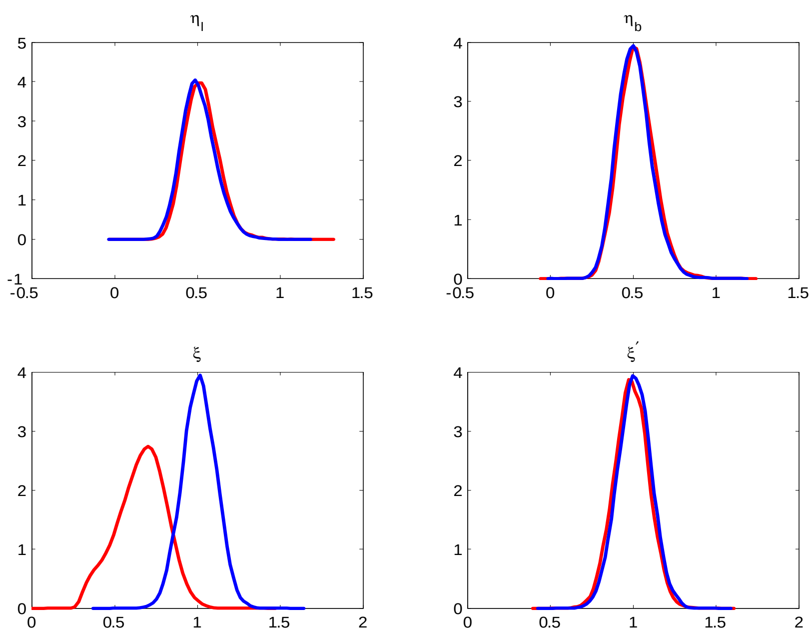
3.3 Prior and posterior densities

In the following graphs we report the prior and posterior densities of selected parameters. The posterior ones are based on 500,000 draws from the Metropolis algorithm and are estimated using a Gaussian kernel. Red lines denote the posterior density while the blue one the prior density.

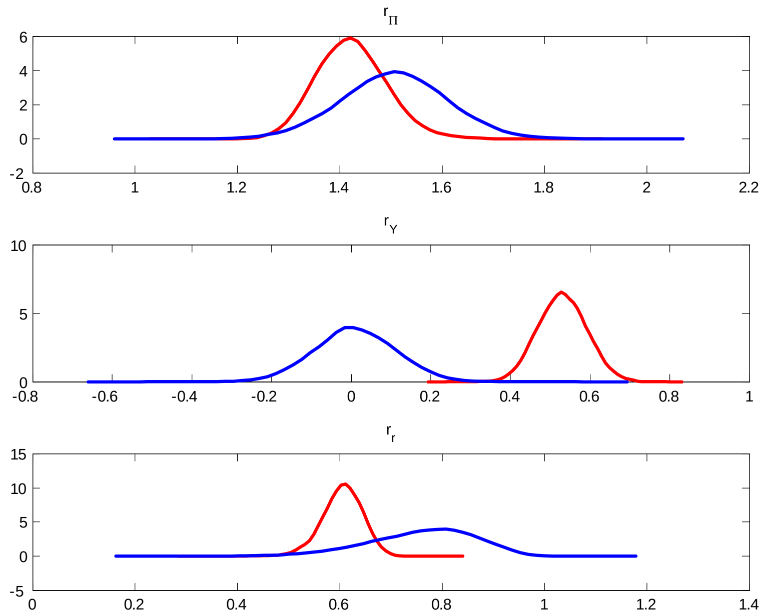


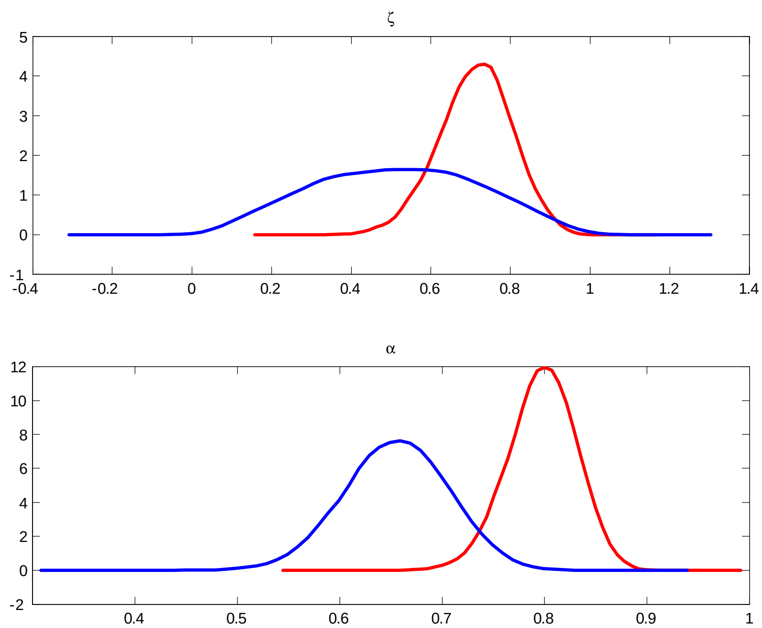


ε
10
8
6
4
2
0
0 0.2 0.4 0.6 0.8 1
ε´
8
6
4
2
0
-2
0 0.2 0.4 0.6 0.8 1
$\phi_{k,c}$
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-10 0 10 20 30
$\phi_{k,h}$
0.2
0.15
0.1
0.05
0
-10 0 10 20 30 40



η_l
5
4
3
2
1
0
-1
-0.5 0 0.5 1 1.5
η_b
4
3
2
1
0
-0.5 0 0.5 1 1.5
ξ
4
3
2
1
0
0 0.5 1 1.5 2
ξ´
4
3
2
1
0
0 0.5 1 1.5 2


r_Π
r_Y
r_r


ζ
α