Appendix to "Housing and Debt over the Life Cycle and over the Business Cycle"

Matteo Iacoviello[*]
Federal Reserve Board

Marina Pavan[†]
Universitat Jaume I & LEE

April 26, 2012

---

[*]Matteo Iacoviello, Division of International Finance, Federal Reserve Board, 20th and C St. NW, Washington, DC 20551. E-mail: matteo.iacoviello@frb.gov.

[†]Marina Pavan, Universitat Jaume I & LEE, Castellón, Spain. E-mail: pavan@eco.uji.es.

1. Appendix A: Computational Details

We solve for the model equilibrium using a computational method similar to the one used in Krusell and Smith (1998). The value and policy functions are computed on grids of points for the state variables, and then approximated with linear interpolation at points not on the grids (with the exception of the policy functions for housing, that are defined only on points on the grid). The algorithm consists of the following steps:

1. Specify grids for the state space of individual and aggregate state variables.

   The number of grid points was chosen as follows: 7 points for the aggregate shock, 3 values for the idiosyncratic shock, 25 points for the housing stock, and 500 points for the financial asset.[1] For aggregate capital, we choose a grid of 15 equally spaced points in the initial range $[0.8K^*, 1.2K^*]$, where K^* denotes the average value of this variable in the simulations. The range is then updated at each iteration consistently with the simulated K, assigning as its boundaries the minimum and the maximum simulated values.

2. Guess initial coefficients $\left\{\omega_i^A\right\}_{A \in \widetilde{A}, i=0,1}$ for the linear functions that approximate the laws of motion of capital and labor:

$$K_t = \omega_0^A + \omega_1^A K_{t-1}, \tag{1}$$
$$L_t = \omega_2^A + \omega_3^A K_{t-1}. \tag{2}$$

   Because factor prices (wages and interest rates) only depend on aggregate capital and labor in equilibrium, this approach is equivalent to assuming that individuals forecast these factor prices using a function of K_{t-1} for each value of the aggregate state A.

3. Starting from age T backward, compute optimal policies as a function of the individual and aggregate states, solving first the homeowner's and renter's problems separately.[2] Notice that the intra-temporal optimal value for labor hours as a function of consumption and productivity shock for ages $a \leq \widetilde{T}$ is the following:[3]

$$l_{a,t} = \bar{l} - \frac{\tau c_{a,t}}{w_t \eta_a z_t} \tag{3}$$

   which allows one to derive consumption before age $\widetilde{T}$ directly from the budget constraint. For the homeowner:

$$c_{a,t} = \frac{w_t \eta_a z_t \bar{l} - R_t b_{a,t-1} + b_{a,t} + (1 - \delta_H) h_{a,t-1} - h_{a,t} - \Psi(h_{a,t}, h_{a,t-1})}{1 + \tau} \tag{4}$$

   so that the per-period utility function for $a \leq \widetilde{T}$ can be transformed as follows:

$$\widetilde{u}(c_{a,t}, h_{a,t}, w_t z_t) = (1 + \tau) \log c_{a,t} + j \log h_{a,t} + \tau \log(\tau / w_t \eta_a z_t). \tag{5}$$

   For the tenant, taking into consideration the intra-temporal condition for optimal house services to rent:

$$c_{a,t} = \frac{w_t \eta_a z_t \bar{l} - R_t b_{a,t-1} + b_{a,t} + (1 - \delta_H) h_{a,t-1} - \Psi(0, h_{a,t-1})}{1 + \tau + j} \tag{6}$$

   so that the per-period utility function for $a \leq \widetilde{T}$ can be transformed as follows:

$$\widetilde{u}(c_{a,t}, p_t, w_t z_t) = (1 + \tau + j) \log c_{a,t} + j \log(j\theta / p_t) + \tau \log(\tau / w_t \eta_a z_t). \tag{7}$$

   As a consequence, the homeowner's dynamic optimization problem entails solving for policy functions for b and h only, while the renter's one consists in solving for b only. The problems of the retired people $(a > \widetilde{T})$ are similar to the above, where we set $\tau = 0$.

---

[1] The upper bound for the housing grid and the lower bound for debt are chosen wide enough so that they never bind in the simulations.

[2] In computation, we exploit the strict concavity of the value function in the choice for assets as well as the monotonicity of the policy function in assets (for the homeowner problem, the monotonocity is for any given choice of the housing stock).

[3] We prevent individuals from choosing negative hours.

4. Draw a series of aggregate and idiosyncratic shocks according to the related stochastic processes. Draw a series of "death" shocks according to the survival probabilities. Use the (approximated) policy functions and the predicted aggregate variables to simulate the optimal decisions of a large number of agents for many periods. In the simulations, we perform linear interpolation between grid points for b', but we restrict the choices of h' to lie on the grid. We simulate $90,000$ individuals for $5,000$ periods, discarding the first 200 periods.[4] Compute the aggregate variables K and L at each t.

5. Run a regression of the simulated aggregate capital and the simulated aggregate labor on lagged aggregate capital, retrieving the new coefficients $\{\omega_i^A\}$ for the laws of motion for K and L. We repeat steps 3 and 4 until convergence over the coefficients of the regressions. We measure goodness of fit using the R^2 of the regressions: they are always equal to 0.997 or higher at convergence for K and around 0.95 for L; the corresponding wage rate and interest rate functions are also very accurate: the R^2 of the regression of the wage rate on aggregate K is 0.999, the R^2 of the regression of the interest rate on aggregate K is 0.992.

---

[4]We enforce the law of large numbers by making sure that the simulated fractions of ages and of labor productivity shocks correspond to the theoretical ones, by randomly adjusting the values of the shocks.

2. Appendix B: Calibrating the Income Process

2.1. The Persistence of Wage Shocks

The (parsimonious) process for individual income productivity that we specify in the model is:

$$\log z_t = \bar{z} + \rho_Z \log z_{t-1} + \sigma_Z \left(1 - \rho_Z^2\right)^{1/2} \varepsilon_t, \quad \varepsilon_t \sim Normal\,(0,1)\,. \tag{8}$$

We want to pick values for ρ_Z and σ_Z that are in line with evidence.

1. Floden and Lindé (2001) estimate an AR(1) process for wages of the form in (8) and estimate (using PSID data covering the 1988-1992 period), after controlling for observable characteristics and measurement error, values of $\rho_Z = 0.91$ (and $\sigma_Z \left(1 - \rho_Z^2\right)^{1/2} = 0.21$, thus implying $\sigma_Z = 0.5$).

2. Heathcote, Storesletten and Violante (2010) estimate an ARMA(1,1) process for wages using PSID data. Their estimate of the autoregressive component is 0.97.

3. Scholz, Seshadri, and Khitatrakun (2006) specify and estimate a model of household log labor earnings (not wages) that controls for fixed effects, a polynomial in age, and autocorrelation in earnings. Their sample is the social security earnings records. Their estimates for married, no college, two-earners are $\rho_Z = 0.70$ (and $\sigma_Z = 0.43$).

2.2. The Change in Volatility

Several studies document the increase in the cross-sectional dispersion of earnings in the United States between the 1970s and the 1990s. This increase is often decomposed into a rise in permanent inequality (attributable to education, experience, sex, etc.) and a rise of the persistent or transitory shocks volatility. Despite some disagreement on the relative importance of these two components, the literature finds that both play a role in explaining the increase in income dispersion.

1. Moffitt and Gottschalk (2008) study changes in the variance of permanent and transitory component of income volatility using data from the PSID from 1970 to 2004. They find that the non-permanent component (transitory) variance of earnings (for male workers) increased substantially in the 1980s and then remained at this new higher level through 2004. They report (see Figure 7 in their paper) that the variance of the transitory component rose from around 0.10 to 0.22 between the 1970s and the 1980s-1990s. This corresponds to a rise in the standard deviation from 0.32 to 0.47. Their estimate of the autocorrelation of the transitory shocks is 0.85.

2. Using PSID data, Heathcote, Storesletten, and Violante (2010) decompose the evolution of the cross-sectional variance of individual earnings over the period 1967-2000 into the variances of fixed effects, persistent shocks, and transitory shocks. They find that the variance of persistent shocks roughly doubles during the 1975-1985 decade.

3. Haider (2001) finds that increases in earnings instability over the 1970s and increases in lifetime earnings inequality in the 1980s account in equal parts for the increase of inequality in the data. To measure the magnitude of earnings instability in year t, he uses the cross-sectional variance of the idiosyncratic deviations in year t. His estimate of ρ_Z is 0.64. He finds that the unconditional standard deviation of the instability component rises from around $0.23 - 0.24$ to about $0.35 - 0.37$ during the 1980s.

4. Krueger and Perri (2006) model log income as an ARMA process of the kind

$$y_t = z_t + \varepsilon_t,\; z_t = \rho_Z z_{t-1} + \sigma_Z \left(1 - \rho_Z^2\right)^{1/2} \varepsilon_t^z,\; \varepsilon_t = \sigma_\varepsilon \varepsilon_t^e \tag{9}$$

where ε_t^e and ε_t^z are $Normal\,(0,1)$. They allow the innovation variances σ_ε and σ_Z to vary by year. They find that the values of σ_Z and σ_ε are respectively 0.42 and 0.28 in 1980, and 0.52 and 0.36 in 2003. Given these numbers, the standard deviation of log income y_t rises by 0.13, from $\sqrt{0.42^2 + 0.28^2} = 0.50$ to $\sqrt{0.52^2 + 0.36^2} = 0.63$.

From this brief review, we conclude that a plausible value for the persistence of the productivity shock is around 0.9. We set the standard deviation of income to be equal to 0.3 in the early part of the sample, which is the lower bound of the estimates reported above. We set the standard deviation to 0.45 in the second part of the sample: a change of 0.15 is in the range of estimates reported by Moffitt and Gottschalk (2008).

3. Appendix C: A Simple Extension with Default

Abstract

This appendix sketches a brief description of an extension of the baseline model in Iacoviello and Pavan (2012) where we allow for mortgage default following housing depreciation shocks.

3.1. Introduction

The following is a brief outline of an extension of the model in Iacoviello and Pavan (2012), where households are allowed to default on their mortgage debt. At any period, indebted households can decide to default on their debt, in which case they lose their house, are banned from borrowing and must become tenants.[5] Default is triggered by shocks to housing depreciation that are large enough to cause leverage individuals to own on their house more than it is worth. The perfectly competitive financial sector cannot discriminate borrowers, that is, lenders cannot apply different borrowing interest rates to different borrowers, and charge the same interest premium to all their debtors in order to break even.

3.2. The model with mortgage default

The environment features the same characteristics as in the baseline model, except for the existence of shocks to the depreciation rate of housing and capital. These shocks are assumed to move one-to-one with the technology shocks: $\delta_{H,t} = \delta_H(A_t)$ and $\delta_{K,t} = \delta_K(A_t)$.[6] As in Iacoviello and Pavan (2011), we adopt the approximate aggregation/bounded rationality approach developed by Krusell and Smith (1997, 1998), and solve for the model equilibrium by forecasting future prices through the first moment of aggregate capital and, in this case, aggregate housing as well. The inclusion of the aggregate housing stock into the set of relevant state variables is necessary in this setup given the existence of shocks to the value of houses, and the need to forecast the interest rate premium as well.[7]

3.2.1. The household's problem

As in the main text, denote $x_t \equiv (z_t, b_{t-1}, h_{t-1}, A_t, H_{t-1}, K_{t-1})$ the vector collecting individual and aggregate state variables. The dynamic problem of an age a household with discount factor β_i can now be stated as:

$$V_a(x_t; \beta_i) = \max_{I^i \in \{I^h, I^r, I^d\}} \{I^h V_a^h(x_t; \beta_i), I^r V_a^r(x_t; \beta_i), I^d V_a^d(x_t; \beta_i)\}$$

where V_a^h, V_a^r and V_a^d are the value functions at age a for owning, renting a house and defaulting respectively, and $I^i = 1$ corresponds to the decision to buy/own, rent or default for $i = h, r$ or d. The value of being a homeowner solves:

$$V_a^h(x_t; \beta_i) = \max_{c_t, b_t, h_t, l_t} \{\lambda_a u(c_t, h_t, \bar{l} - l_t) + \beta_i \chi_{a+1} \sum_{z', A'} \pi_{A,A'} \pi_{z,z'} V_{a+1}(x_{t+1}; \beta_i)\}$$

$$\text{s.t.} \qquad c_t + h_t + \Psi(h_t, h_{t-1}) = y_{at} + b_t - (R_t + \mathcal{I}\{b_{t-1} > 0\} r_t^p) b_{t-1} + (1 - \delta_{H,t}) h_{t-1}$$

$$b_t \leq \min\{m_H h_t, m_Y \Re_t\}, \ c_t \geq 0, \ l_t \in (0, \bar{l})$$

where we use the same notation than in the main paper to denote the transaction costs for housing, etc. The function $\mathcal{I}\{b > 0\}$ is equal to 1 if $b > 0$, i.e. if the household is a net debtor at the beginning of the period. We denote with r_t^p the interest rate premium charged to borrowers. The depreciation rate for housing $\delta_{H,t}$ changes over the business cycle, being higher in the worst recession.

---

[5] In this simple version, the household is banned from borrowing in the default period only, and no credit history is recorded.

[6] In the numerical implementation, capital depreciation is assumed to rise together with housing depreciation to avoid perverse substitution effects between capital and housing investment, which would lead to an increase in aggregate capital when a bad shock to housing hits. Moreover, the numerical implementation assumes that the variance of technology shocks in arbitrarily small, so that the only shocks are effectively the two depreciation shocks.

[7] The typical R^2 of the forecasting equations for K, R and the interest premium is 0.99, 0.995 and 0.99 respectively for the regressions including H. It drops to 0.89, 0.99 and 0.98 when we do not include housing in the forecasting regressions.

As in the benchmark model, the value of renting a house is determined by solving the problem:

$$V_a^r(x_t;\beta_i) = \max_{c_t,b_t,s_t,l_t} \{\lambda_a u\left(c_t,s_t,\bar{l}-l_t\right) + \beta_i \chi_{a+1} \sum_{z',A'} \pi_{A,A'} \pi_{z,z'} V_{a+1}(x_{t+1};\beta_i)\}$$

$$\text{s.t.} \quad c_t + p_t s_t + \Psi(0,h_{t-1}) = y_{at} + b_t - R_t b_{t-1} + (1-\delta_{H,t})h_{t-1}$$

$$b_t \leq 0, \ c_t \geqslant 0, \ l_t \in \left(0,\bar{l}\right), h_t = 0.$$

Households that have a net negative asset position ($b_{t-1} > 0$) at the beginning of the period have the option of defaulting on their debt, losing their house and being only able to rent. The corresponding value is the following:

$$V_a^d(x_t;\beta_i) = \max_{c_t,b_t,s_t,l_t} \{\lambda_a u\left(c_t,s_t,\bar{l}-l_t\right) + \beta_i \chi_{a+1} \sum_{z',A'} \pi_{A,A'} \pi_{z,z'} V_{a+1}(x_{t+1};\beta_i)\}$$

$$\text{s.t.} \quad c_t + p_t s_t = y_{at} + b_t$$

$$b_t \leq 0, \ c_t \geqslant 0, \ l_t \in \left(0,\bar{l}\right), h_t = 0.$$

At the agent's last age, $V_{T+1}(x_{T+1};\beta) = 0$ for any $(x_{T+1};\beta)$.

At any point in time, the following are the forecasting functions:

$$\text{for aggregate capital:} \quad K_t = F^K(K_{t-1}, H_{t-1}, A_t)$$

$$\text{for aggregate labor:} \quad L_t = F^L(K_{t-1}, H_{t-1}, A_t)$$

$$\text{for aggregate housing:} \quad H_t = F^H(K_{t-1}, H_{t-1}, A_t).$$

Moreover, we assume the agents directly forecast the value of the interest rate premium as a function of aggregate capital, housing stock and total factor productivity, $r_t^p = F^p(K_{t-1}, H_{t-1}, A_t)$.[8]

3.2.2. The financial sector with the possibility of mortgage default

In the perfectly competitive financial sector with the option to default, the interest rate on loans is higher than the one on deposits, so that the financial intermediaries' profits are zero. We assume that lenders cannot observe (or face a high cost of observing) the default probability of each individual household or, correspondingly, cannot price discriminate among borrowers and must charge the same interest rate premium r_t^p on every loan.[9] When someone defaults, the financial intermediary retrieves the value of the housing collateral, net of depreciation and transaction costs.

Let's denote with D_{t-1} the aggregate debt at the beginning of period t, of which D_{t-1}^N is the total amount re-paid (not defaulted upon) and D_{t-1}^D is the total amount defaulted, so that $D_{t-1} = D_{t-1}^N + D_{t-1}^D$ at any period. Then a zero profit condition holds such that:

$$D_{t-1} = \frac{(R_t + r_t^p)D_{t-1}^D + (1 - \delta_{H,t} - \Psi(0,H_{t-1}^D))H_{t-1}^D}{R_t}$$

where H_{t-1}^D is the collateral (aggregate value of houses guaranteeing the defaulted debt) repossessed by the financial sector.

Re-arranging, the interest rate premium at any t is then given by

$$r_t^p = \frac{R_t D_{t-1}^D - (1 - \delta_{H,t} - \Psi(0,H_{t-1}^D))H_{t-1}^D}{D_{t-1}^N}$$

---

[8] To the best of our knowledge, Nakajima and Rios-Rull (2005) is the only model to include aggregate risk and default (in the form of consumer bankruptcy) in a heterogeneous agents' equilibrium setting. In their model, however, the assumptions on the timing of the default decision ensure that the prices of loans do not depend on the distribution of agents. We take a different approach and adopt a "bounded rationality" technique to forecast borrowing premia, similar to the one used in Krusell and Smith (1997).

[9] We adopted this modeling strategy for the interest rate premium since it is the most consistent with our setting, in which, as in RBC models in general, interest rates are determined "ex-post" as a function of next period' aggregate shock realization.

One alternative could have been to condition the interest rate premium on the characteristics of the borrower. In that case, though, given the timing assumption of our model, we should have kept track of complex multi-dimensional objects dependent on individual and aggregate variables, and the zero-profit condition would not have been a trivial object to define ex-post.

In the default literature with no aggregate volatility, financial intermediaries commit "ex-ante" to being paid a certain interest rate, so that ex-post profits can be different from zero (Athreya, 2008; Chatterjee et al., 2007; Chatterjee and Eyigungor, 2011).

and is charged to all borrowers, households and firms alike.[10],[11]

3.2.3. Definition of Equilibrium.

We are now ready to define the equilibrium for this economy.

Definition 3.1. *A recursive competitive equilibrium consists of value functions* $\{V_a(x_t; \beta)\}$*, policy functions* $\{I_a^h(x_t; \beta), I_a^r(x_t; \beta), I_a^d(x_t; \beta), h_a(x_t; \beta), s_a(x_t; \beta), b_a(x_t; \beta), c_a(x_t; \beta), l_a(x_t; \beta)\}$ *for each* β*, age and period* t*, prices* $\{R_t\}_{t=1}^{\infty}$*,* $\{r_t^p\}_{t=1}^{\infty}$*,* $\{w_t\}_{t=1}^{\infty}$ *and* $\{p_t\}_{t=1}^{\infty}$*, aggregate variables* K_t, L_t, H_t^o *and* H_t^r *for each period* t*, lump-sum taxes* Γ *and pension* P*, and laws of motion* F^K*,* F^H*,* F^L *and* F^p *such that at any* t*:*

Agents optimize: Given R_t*,* w_t*,* p_t *and* r_t^p *and the laws of motion* F^K*,* F^H*,* F^L *and* F^p*, the value functions solve the individual's problem, with the corresponding policy functions.*

Factor prices and rental prices satisfy:

$$R_t + r_t^p - 1 + \delta_K = \alpha A_t \left(K_{t-1}/L_t\right)^{(\alpha-1)}$$

$$w_t = (1 - \alpha) A_t (K_{t-1}/L_t)^{\alpha}$$

$$p_t = E_t \left(\frac{R_{t+1} - (1 - \delta_H)}{R_{t+1}} \right)$$

and the interest rate premium r_t^p *is determined from the equilibrium condition of the financial sector as above.*

Markets clear:

$$L_t = \int l_a(x_t; \beta) \eta_a z_t \partial \Phi_t \quad \text{(labor market)}$$

$$C_t + H_t - (1 - \delta_{H,t}) H_{t-1} + \Omega_t + K_t - (1 - \delta_{K,t}) K_{t-1} = Y_t \quad \text{(goods market)}$$

where H_t *and* Ω_t *are defined as*

$$H_t = H_t^o + H_t^r = \int I_a^h(x_t; \beta) h_a(x_t; \beta) \partial \Phi_t + \int [I_a^r(x_t; \beta) + I_a^d(x_t; \beta)] s_a(x_t; \beta) \partial \Phi_t,$$

$$\Omega_t = \int \Psi(h_a(x_t; \beta), h_{t-1}) \partial \Phi_t$$

and, by Walras' law, the supply of savings equals total capital.

The government budget is balanced:

$$\sum_{a=1}^{\widetilde{T}} \Pi_a \Gamma = \sum_{a=\widetilde{T}+1}^{T} \Pi_a P.$$

The laws of motion for the aggregate capital, aggregate labor, aggregate housing and interest rate premia are given by

$$K_t = F^K(K_{t-1}, H_{t-1}, A_t), \ L_t = F^L(K_{t-1}, H_{t-1}, A_t)$$

$$H_t = F^H(K_{t-1}, H_{t-1}, A_t), \ r_t^p = F^p(K_{t-1}, H_{t-1}, A_t).$$

---

[10] However, we do not model firms' decision to default. We assume that firms also have to pay the higher interest for borrowing, given that lenders cannot discriminate interest rates on loans.

[11] More precisely, the interest rate premium calculated on the basis of the equilibrium condition is the following:

$$r_t^p = \frac{R_t \int I_a^d(x_t; \beta) b_{t-1} \partial \Phi_t - \int I_a^d(x_t; \beta)(1 - \delta_{H,t} - \Psi(0, h_{t-1})) h_{t-1} \partial \Phi_t}{K_{t-1} + \int (1 - I_a^d(x_t; \beta)) \mathcal{I}(b_{t-1} > 0) b_{t-1} \partial \Phi_t}$$

3.3. Brief outline of numerical implementation

Households perceive that prices depend on the first moment of the aggregate capital and the aggregate housing stock only, and that these variables change over time according to the laws of motion specified above. In particular, agents take their decisions based initially on an arbitrary value of the interest rate premium r^p, and consider the future r^p to be given by a linear function of K, H and A (see Krusell and Smith, 1997).

Given the optimal policy functions solving the individual problem, we simulate the agents' choices and directly compute the interest premium that makes the financial intermediaries' profits to be nul at any period, for a large number of periods.

We then use the obtained time series (of which we discarded the first part) to regress the aggregate variables K_{t+1}, H_{t+1}, L_{t+1} and the premia r^p_{t+1} on constants, K_t and H_t, for each value of the aggregate shock A_t.

We iterate these steps (solution of optimal rules and simulation) until convergence of the parameters in the laws of motion, measuring goodness of fit of the regressions with the implied R^2.

3.4. Results

The model can be used to see how shocks to housing values interact with the mortgage default rate, interest rate, debt and housing stock. To illustrate the main mechanism at work in the model with default, we assume technology shocks away, and solve the model with depreciation shocks for housing and capital only. We fix the labor supply at unity, so that movements in the aggregate capital stock are the only source of movements in output. We choose the model parameters at the values of Table 2 in Iacoviello and Pavan (2011), except the discount rate gap which is 4 percent, and the loan-to-value which is set at 85 percent. The depreciation shocks for housing and capital are set to $\delta_H = 25\%$ and $\delta_K = 13\%$ respectively in the worst state of the world, and to $\delta_H = 15\%$ and $\delta_K = 11\%$ in the next worst case, while $\delta_H = 5\%$ and $\delta_K = 9\%$ in all other states. Recall that the transaction cost to change housing stock is 5 percent, except in the case of default when the defaulting agent can walk away from the debt at no cost.[12]

Figure A.1 illustrates the homeowner's optimal default decision for different combinations of initial house, loan-to-value (LTV) ratio and idosyncratic income shock. In response to a housing depreciation shock that wipes 25% of the house value, homeowners who are characterized by a bad idiosyncratic income realization and by an initial leverage ranging from 68 to 73 percent or higher will choose to default. To consider what this means, assume that the house is worth 100, so that the initial mortgage balance in the house is 68 to 73 dollars. The depreciation shock reduces the value of the house to 75, so "poor" agents who own on their house between $68 - 73$ and higher will choose to default. Notice in the Figure that the bigger is the initial house, the lower is the LTV threshold that triggers default: households with a very high housing stock are more far away from their target level of housing, the default option allows them to save the high transaction costs to pay, so they are willing to default even in the case in which they still have some equity left in the house (after the depreciation shock), provided that the equity in the house is less than the transaction cost.

Figure A.2 shows a simulation of the main macroeconomic variables over 100 model periods. In the bad states of the world, when housing depreciation takes on very large values, interest rate premia reach values of about 1.5 percent, the aggregate default rate rises from 0 to about 10 percent, and the aggregate housing and capital stock persistently decline. Further details on computational results can be obtained from the authors.

---

[12] It would be straightforward to add to the model other penalties for defaulting (income loss, stigma) besides exclusion from the credit market in the current period.

Figures

<div align="center">

Figure A.1: Default Policy in different states of the world

</div>
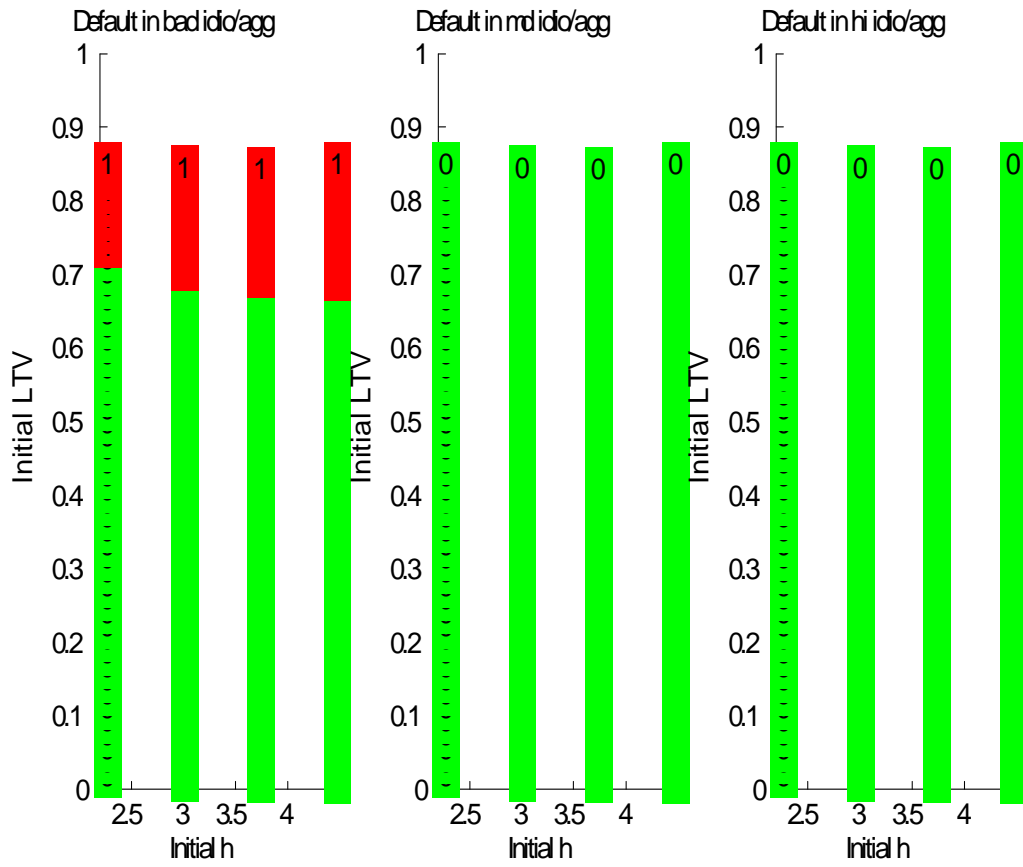
Note: The figure illustrates, for each combination of initial house and LTV, the homeowner's default decision. It is plotted for an impatient agent who is 35 years old. From the left to the right: lowest idiosyncratic and lowest aggregate state; median idiosyncratic and median aggregate state; highest idiosyncratic and highest aggregate state.
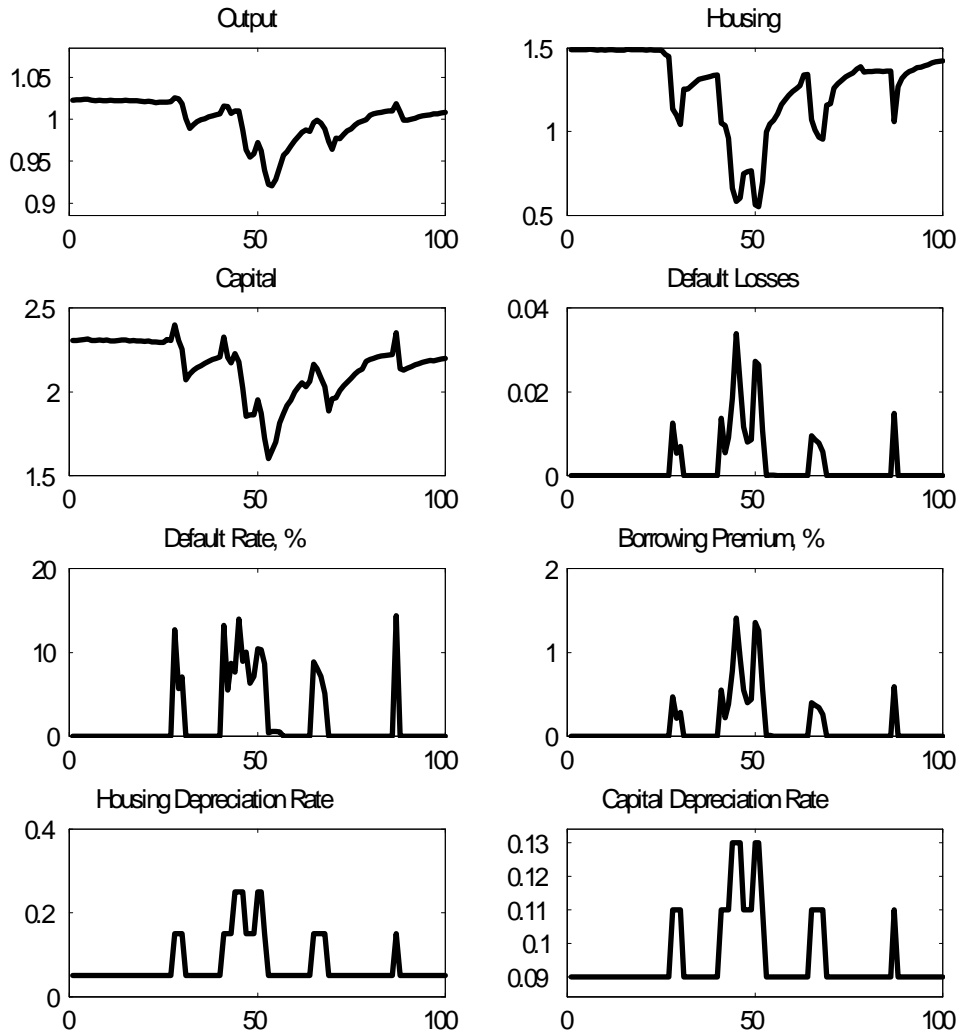
Figure A.2: Macroeconomic variables in default and no–default periods

Note: The figure illustrates a simulation of 100 periods. Average output is normalized to unity. Housing, Capital and Default Losses are expressed as a ratio to average output. Defaults rise in bad states of the world when the housing and capital stock are subject to depreciation shocks.

References

Athreya, Kartik B. (2008) 'Default, insurance, and debt over the life-cycle.' *Journal of Monetary Economics* 55(4), 752–774

Chatterjee, Satyajit, and Burcu Eyigungor (2011) 'A quantitative analysis of the u.s. housing and mortgage markets and the foreclosure crisis.' Working Papers 11-26, Federal Reserve Bank of Philadelphia

Chatterjee, Satyajit, Dean Corbae, Makoto Nakajima, and José-Víctor Ríos-Rull (2007) 'A quantitative theory of unsecured consumer credit with risk of default.' *Econometrica* 75(6), 1525–1589

Floden, Martin, and Jesper Lindé (2001) 'Idiosyncratic risk in the united states and sweden: Is there a role for government insurance?' *Review of Economic Dynamics* 4(2), 406–437

Haider, Steven J (2001) 'Earnings instability and earnings inequality of males in the united states: 1967-1991.' *Journal of Labor Economics* 19(4), 799–836

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante (2010) 'The macroeconomic implications of rising wage inequality in the united states.' *Journal of Political Economy* 118(4), 681–722

Iacoviello, Matteo, and Marina Pavan (2012) 'Housing and debt over the life cycle and over the business cycle.' Unpublished Manuscript

Krueger, Dirk, and Fabrizio Perri (2006) 'Does income inequality lead to consumption inequality? evidence and theory.' *Review of Economic Studies* 73(1), 163–193

Krusell, Per, and Anthony A. Smith (1997) 'Income and wealth heterogeneity, portfolio choice, and equilibrium asset returns.' *Macroeconomic Dynamics* 1(02), 387–422

Krusell, Per, and Anthony A. Smith (1998) 'Income and wealth heterogeneity in the macroeconomy.' *Journal of Political Economy* 106(5), 867–896

Nakajima, Makoto, and José-Víctor Ríos-Rull (2005) 'Default and aggregate fluctuations in storage economies.' In 'Frontiers in Applied General Equilibrium Modeling' in honor of Herbert Scarf (Cambridge University Press)

Scholz, John Karl, Ananth Seshadri, and Surachai Khitatrakun (2006) 'Are americans saving "optimally" for retirement?' *Journal of Political Economy* 114(4), 607–643