

An Equilibrium Model of Lumpy Housing Investment

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We formulate and solve a dynamic general equilibrium model with heterogeneous agents and lumpy housing adjustment at the household level. We use the model to ask a simple question: how does the microeconomic lumpiness of housing adjustment affect the equilibrium dynamic properties of aggregate consumption and investment? Our main conclusion is that lumpiness matters: in particular, lumpiness in housing adjustment (1) reduces the volatility of both housing and business investment; (2) increases the volatility of aggregate consumption; (3) increases the correlation of housing investment with business investment and with GDP. We also show that lumpiness of investment activity at the household level has small but significant aggregate implications, in contrast with the literature that shows that the aggregate effects of lumpy investment at the firm level are negligible. [JEL Classification: E21, E32, E44, D91]

In questo articolo, formuliamo e risolviamo un modello dinamico di equilibrio generale con agenti eterogenei e costi nell'aggiustamento dei beni durevoli (case) da parte delle famiglie. Il modello viene utilizzato per analizzare gli effetti di tali costi sulle dinamiche macroeconomiche di consumo, investimento delle famiglie (case) e investimento delle imprese. Simulazioni del modello mostrano che questi costi sono importanti, in quanto: (1) riducono la volatilità dell'investimento sia delle famiglie che delle imprese; (2) accrescono la volatilità del consumo totale; (3) fanno aumentare la correlazione dell'investimento nel settore delle case sia con l'investimento delle imprese che con il PIL.

1. - Introduction

In this paper, we formulate and solve a dynamic general equilibrium model of housing investment in which housing

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investment is lumpy at the household level. The model feature that contributes to make housing investment lumpy are non-convex transaction costs for housing adjustment. In all other respects, our economy is a simple extension of an otherwise standard heterogeneous-agents, incomplete markets, real business cycle model with aggregate and idiosyncratic shocks in the tradition of papers like Krusell and Smith (1998), Chang and Kim (2007), and Silos (2007).

Our main interest is in understanding how the business cycle properties of the model are affected by the presence of housing when housing adjustment becomes lumpy. In broader terms, we study the implications for aggregate dynamics of an economy in which individuals can accumulate assets characterized by different degrees of liquidity. The agents of our model can hold two assets: capital and housing. Capital can be easily acquired and sold in the market; instead, houses are assets with a limited amount of liquidity: they can be purchased, accumulated or sold, but only subject to various transaction costs.

We characterize stochastic stationary equilibria of our models numerically, and then compare their aggregate properties. Our main results are as follows. Relative to a reference model without lumpy housing adjustment, the model with lumpy housing has the following properties:

1. Because housing wealth is more illiquid, individuals prefer to hold a larger buffer stock of resources when they cannot easily adjust their housing positions. Therefore average savings are higher and the average interest rate is lower in the lumpy housing economy. Relative to an economy without adjustment costs, individuals tend to hold more capital and less homes when homes become more illiquid.

2. Because housing wealth cannot be easily adjusted in response to aggregate and individual shocks, households have fewer adjustment margins in order to smooth fluctuations in non-durable consumption. Hence aggregate and individual consumption are more volatile when housing is lumpy and more illiquid.

3. With lumpy housing, aggregate consumption is more volatile, whereas aggregate housing and non-housing investment are less volatile. Overall, the volatility of total GDP is smaller.

2. - Facts about (Lumpy) Housing

Our interest is in developing a realistic model of lumpiness in housing adjustment where lumpiness stems from transaction costs associated with changing housing consumption.¹ Our notion of realism is a model that is roughly in line with aggregate data on housing and the macroeconomy. For this reason, data from the National Income and Product Accounts of the United States (henceforth NIPA) are our preferred way of measuring the various transaction costs associated with adjusting housing consumption. Below, we review some facts about housing that guide the formulation, calibration and evaluation of our model.

Moving and Transaction Costs. From the old saying that Rome wasn't built in a day to the wealth of available empirical evidence that suggests that households move home only infrequently, several pieces of evidence point out to the fact that, when households adjust their housing consumption, they often do so only by paying substantial transaction costs. Quigley (2002) reviews the different forms that these transaction costs take. These costs include the direct costs of moving, the search costs of finding a new home, the costs of matching a borrower with a lender, the legal and administrative costs of closing the sale of a home, and the psychic costs of moving. Ghent (2007) surveys the empirical evidence: her numbers suggest that the transaction costs associated with changing housing consumption, whenever the change takes the form of a move, range around 5 to 10 percent of the value of the house.

Renovation Costs. Households can adjust their housing consumption in continuous amounts by letting their home depreciate (for instance, by not undergoing repair and maintenance), or by making some home improvements which add value to the existing stock (for instance, adding a new heating system, remodeling

¹ We define lumpy as large and infrequent. This behavior can be easily reconciled with non-convex transaction costs of housing adjustment. Of course, information processing costs, life-cycle motives and other frictions might also contribute to make investment in housing lumpy at the micro level (even in absence of non-convex transaction costs). We do not explore these possibilities here.

a bathroom, painting the walls — activities that are counted as residential investment in GDP). Typically, improvements and renovations add value to the home in a one for one fashion, although anecdotal evidence suggests that the price of a home improvement project may not always be recouped. For instance, Remodelling Magazine “Cost vs. Value Report,” estimates that home remodelling projects typically recoup about 80 percent of the cost.²

Housing and Housing Transaction Costs in National Income and Product Accounts. Housing contributes to the nation’s gross domestic product in two basic ways: directly, through residential fixed investment, and indirectly, through consumption spending on housing services. Residential investment accounts for 5 percent of GDP, housing consumption for 10 percent of GDP. For this reason, the housing sector accounts for roughly 15 percent of a nation’s gross domestic product.

Residential investment includes construction of new single-family and multi-family structures, residential remodelling, production of manufactured homes, and brokers’ fees. Consumption spending on housing services includes rents paid by tenants and the imputed value of housing services to home owners.

The United States National Income and Product Accounts break residential investment down into various sub-categories. The major component counts the value of new additions to the existing housing stock: this is called “permanent site” residential investment, and accounts for roughly two thirds of total residential investment. In addition to this category, home improvements account for about 20 percent, and brokers’ commissions account for about 10 to 15 percent.

Each year, brokers’ commissions on the sales of new and existing homes are counted as part of a nation’s gross domestic product in the residential fixed investment category. In 2005, for instance, total brokers’ commissions were 87 billion dollars, about 15 percent of total residential fixed investment (slightly less than 1 percent of total GDP in the US). In practice, these brokers’ commissions are indirectly calculated imputing a commission on

² The report was published by Hanley-Wood LLC in 2002.

the total value of home sales in a particular year. In 2005, the value of existing home sales was 1,279 billion dollars, the value of new home sales was 307 billion dollars. Given a total value of 1,784 billion dollars, brokers' commissions amount to roughly $87/1784 \approx 5\%$ of the value of transactions. This number would imply a market price of 5 percent in computing the transaction costs associated with changing housing consumption. We use the italic above because our reading of several pieces of information, however, suggests that the 5 percent rule might exaggerate the average market price of buying and selling a home. First, brokers' commissions are not computed using independent information; instead, they are calculated under the assumption that these commissions are 6 percent of the value of existing home sales and 4 percent of new home sales. However, Bureau of Labour Statistics (BLS) data on income of real estate brokers' do not match up with Bureau of Economic Analysis (BEA) data: according to the BLS, there are about 432,000 real estate sales agents in the United States, and each made on average an income of 40,000 dollars in 2005.³ If these numbers are correct, totals brokers' income is 17 billion dollars, that is 5 times smaller than the figure imputed from the BEA.⁴

Summary. We conclude this brief and selected survey by summarizing that: (1) large changes in housing consumption involve large transaction costs, since they are typically associated with a move that involves selling an old home and buying a new one; (2) small changes in housing consumption do not involve major transaction costs, since the cost of renovations is often recouped; (3) the cost of large changes is typically linear in the value of the house: the total (non-market and market) transaction cost is around 10 percent; the market transaction cost (which excludes the time spent searching a new house, the emotional distress associated with a move, and so on) is smaller, probably in the 3 to 6 percent range.

We use the numbers above as an input in the calibration of our model. Before doing so, we present our setup in the next section.

³ See <http://www.bls.gov/oco/ocos120.htm>.

⁴ Under the assumption that half of the agents' commission goes to the firm they work for, commissions calculated by BEA in the National Income and Product Accounts are still 2.5 times larger than those produced by the BLS statistics.

3. - The Model Economy

Our benchmark economy is a version of the stochastic growth model with heterogeneous consumers, extended to allow for housing investment, collateralized borrowing and housing adjustment costs. There is a continuum of infinitely lived agents who have an exogenously specified stochastic endowment process. Time is discrete. Each consumer has preferences over a non-durable consumption good, durable housing and leisure.

At each point in time, agents differ in two respects:

1. Their realized labour productivity;
2. Their degree of patience: a recent branch of this literature suggests that preference heterogeneity may be an important source of wealth inequality. This is motivated by the finding that similar households hold very different amounts of wealth. For example, Venti and Wise (2001) study wealth inequality at the outset of retirement among households with similar lifetime earnings and conclude that “the bulk of the dispersion must be attributed to differences in the amount that households choose to save”.⁵

There are no state contingent markets for hedging against idiosyncratic risk, and only self-insurance through a risk-free bond is possible. Agents can borrow up to a fraction of their housing wealth, and incur a cost in adjusting the housing stock. Finally, aggregate uncertainty is introduced in the form of a shock to total factor productivity. Hence the model uses as inputs the exogenous aggregate and idiosyncratic uncertainty, and delivers as output the endogenously derived dynamics of housing and financial investments over the business cycle.

3.1 *The Household Problem*

Let \bar{l} denote each agents' total time endowment. Households derive utility from leisure ($\bar{l} - l$), non-durable consumption c , and

⁵ KRUSELL P. - SMITH A.A.J. (1998) also explore a heterogeneous-agents setting with preference heterogeneity which, unlike a benchmark model with a single discount factor, replicates some of the key features of the observed data on the distribution of wealth.

service flows from housing, which are assumed to be proportional to the stock h .

They maximize their expected lifetime utility⁶

$$E_0 \sum_{t=0}^{\infty} \beta_t^t u(c_t, h_t, l_t)$$

where β_t is the household specific discount factor, $\beta_t \in (0, 1)$, and E_0 denotes expectations at $t = 0$. We refer to households with a lower value of β as impatient. The discount factor is deterministic, and does not vary over time. In the numerical experiments below, we assume that households are either born impatient or patient. Heterogeneity in discount factors allows the model matching the wealth distribution better than in a model without these features.

The per-period utility function is additively separable in its arguments, and takes the following logarithmic formulation:

$$u(c_t, h_t, l_t) = \log c_t + j \log h_t + \tau \log(\bar{l} - l_t)$$

with $j, \tau > 0$.

Each unit of time endowment supplied in the labor market is paid at the wage rate w_t . At each t , households receive a shock to their efficiency units of labour, $z \in \tilde{Z} \equiv \{z^1, \dots, z^n\}$. The shock follows a Markov process with transition matrix $\pi_{z, z'} = \Pr(z_{t+1} = z' | z_t = z)$, $\pi_{z, z'} > 0$ for every $z, z' \in \tilde{Z}$, with

$$\sum_{z'} \pi_{z, z'} = 1$$

for every $z \in \tilde{Z}$. By a law of large numbers, π also represents the fraction of agents experiencing a transition from z to z' between any two periods, with z and $z' \in \tilde{Z}$. Let Π be the unique stationary distribution associated with the transition probability π . Again, by

⁶ Throughout the paper, lowercase variables refer to individual variables, uppercase variables to aggregate variables.

a law of large numbers, at each period there are $\Pi(z)$ agents characterized by labour productivity z . The total amount of labour efficiency units

$$\sum_{i=1}^n z^i \Pi(z^i)$$

is constant and normalized to one for convenience.

Households can buy and sell only one bond, b , which pays a gross risk-free interest rate of R_t in period t . For convenience, let positive amounts of this bond denote a net debt position.⁷ Housing wealth can be used as collateral for borrowing. At any period, the maximum net debt that households can incur is a fraction m of the housing stock:

$$b_t \leq mh_t$$

It is implicit here that there exists an intermediary financial sector which collects deposits from some households, and lends both to other households and to firms.

Each agent starts out with initial conditions (b_0, h_0) . Given that there are no state contingent markets for the individual shocks, the agent is able to smooth consumption only by adjusting the level of financial stock and housing stock over time.

3.2 *The Environment*

The goods market is perfectly competitive and characterized by constant returns to scale, so that without loss of generality we can consider a single, representative firm only. The good is produced according to the Cobb-Douglas technology:

$$Y = AK^\alpha L^{1-\alpha}$$

⁷ We therefore refer to b as financial liabilities (or net debt), and correspondingly to $-b$ as financial assets (or net assets).

where L and K denote aggregate labour and aggregate capital respectively, $\alpha \in (0, 1)$ is the capital share of aggregate income, while $A \in \tilde{A} \equiv \{A^1, \dots, A^s\}$ represents the stochastic shock to total factor productivity. The aggregate shock is assumed to follow a finite-state Markov process with transition matrix $\pi_{A,A'} = Pr(A_{t+1} = A' | A_t = A)$, with $\pi_{A,A'} > 0$ for every $A, A' \in \tilde{A}$, and

$$\sum_{A'} \pi_{A,A'} = 1$$

for every $A \in \tilde{A}$.

We identify higher or lower aggregate productivity with booms and recessions over the business cycle, and we will analyze the cyclical properties of housing and financial investment.

The economy-wide feasibility constraint requires that at each period t total production of the good, Y_t , corresponds to the sum of aggregate consumption C_t , investment in the stock of aggregate capital K_t , investment in the stock of aggregate housing H_t , and the total transaction costs incurred for adjustments to the housing stock, $\Omega(H_t, H_{t-1})$:

$$(1) \quad C_t + H_t - (1 - \delta_H)H_{t-1} + \Omega(H_t, H_{t-1}) + K_t - (1 - \delta_K)K_{t-1} = Y_t$$

with δ_H and δ_K denoting the depreciation rates of housing and capital, respectively.

We assume that there is no government, nor supply or demand of bonds from abroad. Hence the net supply of financial assets in this economy must be equal to the aggregate level of physical capital, K_t . Factor prices will be determined in equilibrium by the optimization conditions of the representative firm, which maximizes its profits.

3.3 *The Equilibrium*

Denote with $\Phi_t(z_t, b_{t-1}, h_{t-1}; \beta)$ the distribution over productivity shocks, asset holdings, housing wealth, and discount factors in

period t .⁸ Without aggregate uncertainty, the economy would be in a stationary equilibrium, with an invariant distribution Φ and constant prices. Given aggregate volatility however, the distribution Φ will change over time, depending on the evolution of aggregate shocks and the heterogeneity of individual states at any period.

When solving their dynamic optimization problem, agents need to predict the future wage and interest rate. The latter depend on the future productivity shock and aggregate capital-labour *ratio*, which in turn is determined by the overall distribution of individual states.⁹ As a consequence, the distribution $\Phi_t(z_t, b_{t-1}, h_{t-1}; \beta)$ (and its law of motion) is one of the aggregate state variables agents need to know in order to make their decisions (together with total factor productivity). This distribution is an infinite-dimensional object, and its law of motion maps an infinite-dimensional space into itself, which imposes a crucial complication for the solution of the model economy. Indeed, it is impossible to directly compute the equilibrium for such an economy. We thus adopt the computational strategy of Krusell and Smith (1998) and assume that only the first moments of the distribution Φ are sufficient to forecast future prices. Krusell and Smith's approach can be seen as a mere computational device to solve for an "approximate" equilibrium in this kind of models. In a different interpretation, agents could be thought of as having "partial information", or being characterized by "bounded rationality". In any case, Krusell and Smith (1998) show that their methodology is accurate enough so to have very small forecasting errors and an "approximate" equilibrium that is very close to the exact one.

We can write the household problem in recursive formulation. The agent's individual state variables are the productivity shock z_t , the net liabilities position b_{t-1} , and the stock of housing wealth h_{t-1} owned at the beginning of period t . In the spirit of Krusell

⁸ The marginal distribution of Φ_t with respect to z_t is Π , by definition.

⁹ In an economy with heterogeneous agents, people have different incomes and different propensities to save out of their wealth, so that the entire wealth distribution (together with the optimal policy functions) is needed to predict the future aggregate capital-labour *ratio*, and thus the interest rate such that firms' profits are maximized.

and Smith (1998), agents need only forecast future aggregate capital stock and aggregate housing stock in order to predict the next period' wage and interest rate. They observe aggregate housing wealth H_{t-1} and aggregate capital K_{t-1} at the beginning of period t , and approximate the evolution of each of these variables and of aggregate labour with a linear function that depends on the aggregate shock A_t . Denote with $x_t \equiv (z_t, b_{t-1}, h_{t-1}, A_t, K_{t-1}, H_{t-1})$ the vector of individual and aggregate state variables.¹⁰ In recursive form, the dynamic problem of a household with discount factor β_i can be stated as follows:

$$\begin{aligned}
 V(x_t; \beta_i) &= \max_{c_t, b_t, h_t, l_t} \left\{ u(c_t, h_t, l_t) + \beta_i \sum_{z_{t+1}, A_{t+1}} \pi_{A_t A_{t+1}} \pi_{z_t z_{t+1}} V(x_{t+1}; \beta_i) \right\} \\
 (2) \quad \text{s.t.} \quad & c_t + h_t + \Psi(h_t, h_{t-1}) = w_t z_t l_t + b_t - R_t b_{t-1} + (1 - \delta_H) h_{t-1} \\
 & b_t \leq m h_t, c_t \geq 0, l_t \in (0, \bar{l}) \\
 & (K_t, H_t, L_t) = F(K_{t-1}, H_{t-1}; A_t)
 \end{aligned}$$

where F is a linear function in K_{t-1} and H_{t-1} , whose parameters depend on the aggregate shock A_t , and denotes the law of motion of the aggregate states, which agents take as given. The agent sustains an additional cost, proportional to his initial housing stock, only if the adjustment in housing is big enough:

$$\Psi(h_t, h_{t-1}) = \psi h_{t-1} \quad \text{if} \quad \frac{h_t - h_{t-1}}{h_{t-1}} > \phi_h$$

with $\psi, \phi_h \in (0, 1)$. No expense is incurred if the proportional change in housing is lower than ϕ_h . In the discussion below, we will present results for alternative calibrated values of the parameters and ψ and ϕ_h .

¹⁰ We present the ‘‘approximate’’ recursive formulation in which the aggregate state variables are represented by the economy capital and housing stocks. As described in the text, in the ‘‘true’’ definition of the household dynamic problem the entire distribution Φ_t is an argument of the value function.

We are now ready to formally define the equilibrium for this economy.

Definition 3.1. A recursive competitive equilibrium is a value function $\{V(x_t; \beta)\}_{t=0}^{\infty}$, policy functions $\{h(x_t; \beta), b(x_t; \beta), c(x_t; \beta)\}_{t=0}^{\infty}$ for each β , prices $\{R_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$, aggregate variables K_t, L_t , and H_t for each period t , and a law of motion $F(K, H; A)$ such that:

1. *Agents optimize:* Given R_t, w_t , and the law of motion F , the value functions are solution to the individual's problem, with the corresponding policy functions;

2. *Prices are determined competitively at any t :*

$$R_t - 1 + \delta_K = \alpha A_t (L_t / K_{t-1})^{1-\alpha}$$

$$w_t = (1 - \alpha) A_t (K_{t-1} / L_t)^\alpha$$

3. *Assets and Labour Markets clear at any t :*

$$\int b(x_t; \beta) \partial \Phi_t(z_t, b_{t-1}, h_{t-1}; \beta) + K_t = 0$$

$$L_t = \int l(x_t; \beta) z_t \partial \Phi_t(z_t, b_{t-1}, h_{t-1}; \beta)$$

and as a consequence the goods market satisfies the resource feasibility constraint (1).

4. *The law of motion for aggregate capital, housing wealth and labour is given by*

$$(K_t, H_t, L_t) = F(K_{t-1}, H_{t-1}; A_t)$$

Notice that individual labour supply is an intra-temporal decision, and the first order conditions that characterize the optimal $l(x_t; \beta)$ are the following:

$$\frac{w_t z_t}{c(x_t; \beta)} = \frac{\tau}{\bar{l} - l(x_t; \beta)} \quad \text{if } l(x_t; \beta) > 0$$

$$\frac{w_t z_t}{c(x_t; \beta)} < \tau \quad \text{if } l(x_t; \beta) = 0$$

Appendix A at the end provides the details on the computational approach used to solve for the model equilibrium.

4. - Calibration

Our model period is assumed to be a year. We assume that 50 percent of households have a discount factor of 0.95, and 50 percent of households have a discount factor of 0.97. The high discount factor pins the average real interest rate down to an average value around 3 percent, as in the data. The low discount factor is in the range of estimates in the literature, see for instance Hendricks (2007) and references therein.

We set the capital share $\alpha = 0.33$ and its depreciation rate $\delta_K = 0.10$. In all the economies we consider, these values yield average capital to output *ratios* around 2.6 and average investment to output *ratios* around 25% on an annual basis.¹¹

We set the weight on housing in the utility function $j = 0.125$, and the depreciation rate for housing $\delta_H = 0.04$. These values yield average housing capital to output *ratios* of around 1.3 and average housing investment to output *ratios* around 5% on an annual basis. All these values are roughly in accordance with the National Income and Product Accounts and the Fixed Assets Tables.

We set $\tau = 1$ and the total endowment of time $\bar{l} = 2.06$: these two parameters imply that average time worked (around unity) is slightly less than half the available time. As explained in King and Rebelo (2000), this is equivalent to assuming a labour supply elasticity of around unity.

The business cycles features that our model aims at reproducing refer to the US economy. As is well known, the US economy has experienced a substantial decline in the volatility of most macroeconomic aggregates post 1980s, which makes it hard to use a simple summary statistics for the entire post-world-war

¹¹ Our definition of output excludes the value of imputed rents on housing services, which account for about 10% of GDP in the United States.

II period. For this reason, the set of business cycle facts that our model aims at explaining correspond to the US for the period 1985-2005. Our calibration of the aggregate shock is meant to reproduce a standard deviation of output roughly in line with the data for the period 1985-2005. For the aggregate productivity shock A_t we use a Markov-chain specification with five states to match the following first-order autoregressive representation for the logarithm of total factor productivity

$$\log A_t = \rho_A \log A_{t-1} + \sigma(1 - \rho_A^2)^{1/2} \varepsilon_t, \quad \varepsilon_t \sim Normal(0,1)$$

We set $\rho_A = 0.90$ and $\sigma_A = 0.008$.

In a similar vein, we specify the idiosyncratic labour efficiency shock using a three-state Markov chain to match:

$$\log z_t = \rho_Z \log z_{t-1} + \sigma_Z(1 - \rho_Z^2)^{1/2} \varepsilon_t, \quad \varepsilon_t \sim Normal(0,1)$$

We set $\rho_Z = 0.925$ and $\sigma_Z = 0.30$. Our calibration of the idiosyncratic shock is meant to proxy for a reasonable degree of income uncertainty at the household level. The numbers we choose are in line with several microeconomic studies (for instance, see Aiyagari, 1994 and references therein).

Finally, we specify the maximum loan-to-value *ratio* m at 0.95.

5. - Solution

We numerically solve for the model equilibrium using a computational method similar to the one used in Krusell and Smith (1998). The individual's value and policy functions are computed on grids of points for the state variables, and then approximated with linear interpolation at points not on the grids. See the Appendix for details. In the simulations we include 12,000 agents and 10,000 periods; we discard the first 200 periods. Typically, starting from any initial wealth distribution (we choose

one in which agents hold the same level of assets), the stationary equilibrium is reached in about 100 model periods.

6. - Model Results

We now describe the main properties of our model. In order to analyze how lumpy housing investment affects the economy business cycle properties, we compare the following three economies:

1. *Model A*: In the baseline model, individuals can accumulate and decumulate housing without paying any transaction cost. Housing is fully liquid ($\psi = 0$).

2. *Model B*: Adjustment of housing is free if net housing investment does not exceed 3% of the beginning of period stock ($\phi_h = 0.03$), reflecting the assumption that renovations are a way to adjust housing consumption by small amounts without incurring in transaction costs. Otherwise, each household pays a 3 percent transaction cost proportional to the value of the initial stock ($\psi = 0.03$).

3. *Model C*: Any adjustment of housing incurs a 3 percent transaction cost proportional to the value of the initial stock ($\phi_h = 0$, $\psi = 0.03$).

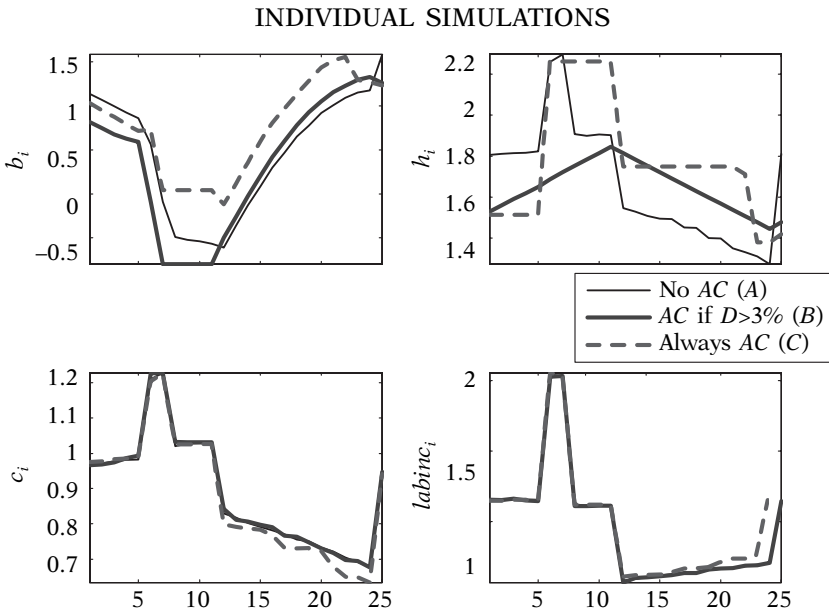
It is obvious that, in moving from *model A* to *model C*, housing becomes more and more illiquid. Individual simulations for 25 period/years are shown in Graph 1. The figure plots financial liabilities (b_i), housing stock (h_i), consumption (c_i) and labour income ($labinc_i$) of a randomly chosen patient agent that we follow for about 25 periods. This agent starts with borrowing around 1 and housing holdings around (depending on the model) 1.5, so that the borrowing constraint (which allows to borrow up to 95 percent of the value of the house) is not initially binding.¹² Initially, individual income is around average, and it rises in period 5, thus leading to a drop in borrowing and a rise in housing and non-

¹² The first period that we plot in the Graph does not coincide with the first period in the simulation: for this reason, the holdings of b and h are not necessarily identical across the three models in the first period plotted.

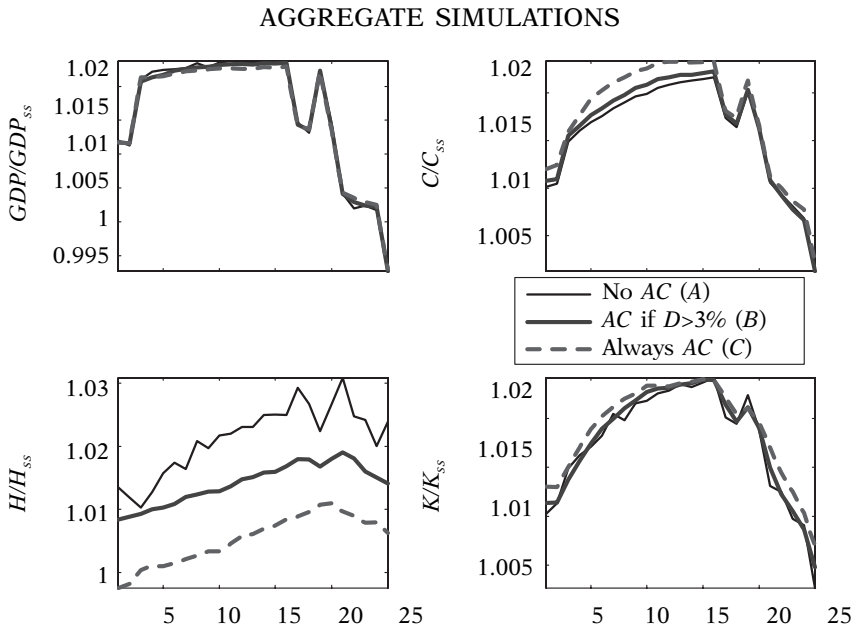
housing consumption. In period 8, however, the agent is hit by a negative idiosyncratic income shock that persists for a while. In the model without housing adjustment costs (*model A*), the agent finds it optimal to reduce consumption, housing and non-housing wealth: in particular, the adjustment in housing is large and immediate. In the models when housing can be adjusted only at some cost, most of the adjustment falls on consumption: if housing can be adjusted only within the 3% band, the adjustment is slow and gradual (*model B*); if all adjustments are costly, the agent reduces its housing consumption only after the bad shock has persisted for a sufficiently long time (*model C*): in the case of *model C*, the agent hits the borrowing constraint around period 22, and is forced to cut back on housing and borrowing at the same time, so his consumption drops substantially.

The same logic applies to aggregate shocks as well. The main difference is that aggregate shocks (being common to every agent) also impact on the equilibrium interest rate, and that aggregate shocks are also smaller in their average size. Graph 2 plots the

GRAPH 1



GRAPH 2



main macro variables (expressed as a *ratio* to their long-run, steady state model average, denoted with the *ss* subscript) for a typical simulation that lasts for 25 periods: here, the economy experiences a prolonged expansion which lasts until around period 17 (total GDP is about 2% above its mean during the expansion). As housing becomes costlier to adjust (that is, as we move from *model A* to *model C*), the stocks of housing and business capital are smoother and less volatile, whereas consumption becomes slightly more volatile.

In Tables 1 and 2 we summarize the quantitative differences across the three models that we compare, and we relate them to their data counterparts. Table 1 presents the statistics computed from the US data for the period 1985-2005. As the data Table shows, investment (consumption) is more (less) volatile than GDP. Three interesting properties of housing investment are that (1) housing investment is procyclical; (2) housing investment leads GDP; (3) housing investment is more volatile than business investment.

TABLE 1

CYCLICAL STATISTICS OF THE US ECONOMY 1984-2005

Variable	sd%	sd/sd(Y)	correlation of Y w/		
			$x(t-1)$	x	$x(t+1)$
<i>GDP</i>	1.21	1.00	0.92	1.00	0.92
<i>C</i>	0.77	0.63	0.82	0.94	0.86
<i>IH</i>	5.03	4.16	0.71	0.64	0.48
<i>IK</i>	4.22	3.48	0.76	0.85	0.84
<i>Debt</i>	1.33	1.10	0.02	0.04	0.00

MEASURES OF INEQUALITY

	Variable	1998	
	Gini Wealth	0.80	
	Gini Income	0.55	
	Gini Earnings	0.61	

Source: BUDRÍA S. *et al.*, 2002.

Table 2 displays the baseline properties (standard deviation and cross-correlation of aggregate output) of the three models.

In all the models, as in the US data, consumption, business investment and housing investment are procyclical. Aggregate consumption is smoother than aggregate output: its relative standard deviation is around 0.8. As in many incomplete markets models, individual consumption is more volatile than aggregate consumption. For instance, the standard deviation of the individual consumption growth rate is around 0.10, a number which is much higher than what a complete markets model would predict. Using the Consumption Expenditure Survey data, Bray, Constantinides, and Geczy (2002) find that the standard deviation of quarterly consumption growth is about 0.063 for households with positive assets. If quarterly consumption growth is i.i.d., this corresponds to a standard deviation of annual consumption growth of 0.126. In addition, the standard deviation of individual consumption growth is less than half the standard deviation of individual income growth (0.23).

TABLE 2

PROPERTIES OF THE MODEL

	<i>Model A</i> No AC	<i>Model B</i> 3% band, no AC	<i>Model C</i> 3% AC
St.dev. <i>GDP</i>	1.21	1.19	1.16
St.dev. <i>C</i>	0.93	0.96	1.05
St.dev. <i>IH</i>	9.87	3.18	2.17
St.dev. <i>IK</i>	2.68	2.01	2.02
St.dev. <i>Hours</i>	0.23	0.2	0.21
St.dev. <i>Debt</i>	1.19	0.59	1
Corr <i>C, GDP</i>	0.95	0.95	0.94
Corr <i>IK, GDP</i>	0.65	0.88	0.91
Corr <i>Hours, GDP</i>	0.74	0.68	0.41
Corr <i>Debt, GDP</i>	0.27	-0.01	-0.04
Corr <i>IH, IK</i>	-0.45	0.3	0.47
Corr <i>IH, GDP(-1)</i>	0.57	0.81	0.72
Corr <i>IH, GDP</i>	0.31	0.61	0.68
Corr <i>IH, GDP(+1)</i>	0.22	0.54	0.62
Average <i>Debt/Y</i>	0.59	0.56	0.57
Std ($\Delta \log c_i$)	0.103	0.105	0.106
Std ($\Delta \log wl_i$)	0.23	0.23	0.23
Gini Wealth	0.66	0.66	0.67
Gini Consumption	0.15	0.15	0.15
Gini Labor Income	0.22	0.22	0.22
Average <i>GDP</i>	1.6107	1.6112	1.6143
Average <i>K</i>	4.1251	4.1288	4.1401
Average <i>H</i>	2.0612	2.0419	2.0555
Average Wealth	6.1863	6.1707	6.1956

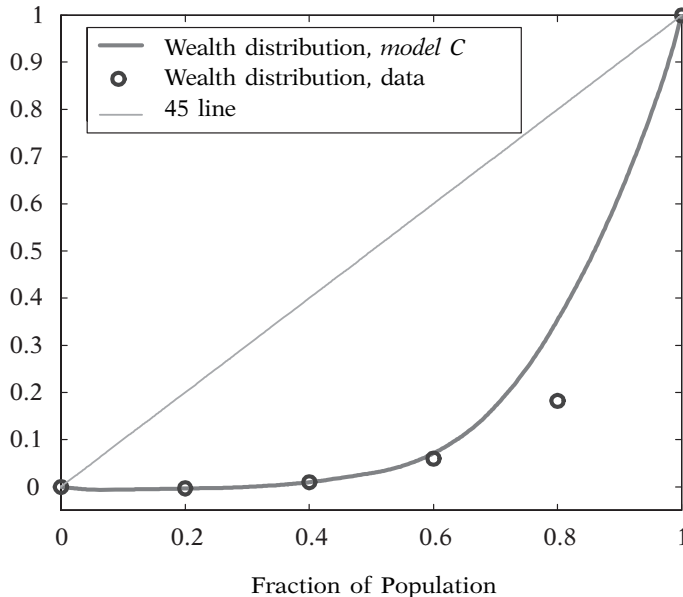
Note: Results with unfiltered variables. (Similar results obtain using *HP*-filter for the model variables).

We conclude this section by remarking the success of our model in matching important elements of the wealth distribution in the data. Graph 3 shows the Lorenz curves for total wealth (the sum of housing and net financial assets) in *model C* and the data.¹³

¹³ We choose *model C* because *model C* is the pure lumpiness model and has some microeconomic appeal. However, the wealth distributions implied by *model C* were nearly identical to those of *model A* and *model B*, thus suggesting that the main differences across models have to do with their business cycle properties, rather than their implied properties for the distribution of wealth.

In our model economy, small differences in the discount factors imply that impatient agents (who are 50 percent of the population) end up accumulating very little wealth besides the required down payment in their home, so they hold close to zero wealth; as a consequence, patient agents hold lots of the wealth in the economy, although bad and persistent income shocks can force some of them to hold very small amounts of it. As in the data, 60 percent of the population holds less than 10 percent of total wealth, although our model overpredicts the share of wealth held by the second quintile. As a consequence, the Gini coefficient for total wealth in our models is slightly less than in the data (0.67 against 0.80). Not reported in the figure, both in our model and in the data the distribution of total wealth is more skewed than the distribution of housing wealth, although our model predicts that housing wealth is much less concentrated than in the data

GRAPH 3

LORENZ CURVES FOR WEALTH HOLDINGS, *MODEL C* AND DATA

(the Gini coefficient for housing wealth is 0.2 in the model, against 0.6 in the data).¹⁴

In all models, it is possible to construct a measure of aggregate mortgage debt by integrating across the negative positions of all indebted agents: this variable amounts to about 60 percent of GDP in all the economies we consider. A large fraction of this debt (about 90 percent) is held by the impatient agents: in this sense, impatient agents are an important element of the model since they allow reproducing the distribution and the composition of aggregate wealth better than models based on agents who are *ex-ante* identical.¹⁵

Volatility of Aggregate Housing Investment. In *model A* agents smooth their consumption over time by freely adjusting both their housing stock and their financial assets. Housing and non-housing capital are very close substitutes, because they both offer the individual an instrument to smooth consumption over time, and do not differ in their degree of liquidity.

When housing becomes more costly to adjust, agents do not change their housing position often, but smooth consumption through buying and selling the financial asset (of which they now need to keep a higher stock). Obviously, this behaviour is more pronounced in *model C* relative to *model B*. Therefore, in terms of volatility of aggregate housing investment, *model A* is the one that delivers the largest volatility (equal to about 8 times that of GDP), followed by *model B* (2.7) and *model C* (1.9).

Intuitively, this result is not surprising, since it is standard extension of partial equilibrium reasoning applied to a general equilibrium model. However, the result stands in sharp contrast with some of the literature that models non-convex costs of investment on the side of the firm, and finds that large and infrequent adjustment of capital on the firm side has no effect on the behaviour of aggregate variables (as in the work of Aubhik Khan and Julia Thomas). Our intuition for the main difference

¹⁴ The Gini coefficients for total wealth and housing wealth in the data were based on the studies of BUDRÍA S. *et AL.* (2002) and DIAZ A. - LUENGO-PRADO M. (2006) respectively.

¹⁵ A similar point is also made by HENDRICKS L. (2007).

between this literature and our results goes as follows: when capital adjustment costs are paid by the firm, adjustments in prices work in a way to smooth household consumption in spite of large lumpiness in investment at the firm; instead, in our case, when adjustment costs are paid by the household, fluctuations in prices are not sufficient to restore smooth aggregate consumption profiles, so that the macroeconomic aggregates do depend on lumpy housing investment.¹⁶

Another interesting property of *models B* and *C* is that the volatility of housing investment goes hand in hand with the volatility of fixed investment. Since buying and selling housing more frequently entails a higher level of financial activity, lower housing adjustment costs imply higher volatility both of housing investment and of business investment.

Volatility of Other Aggregates. *Model A* delivers the largest volatility of housing investment, but delivers the smoothest profile for aggregate consumption. The relative standard deviation of aggregate consumption computed in *model A* is 0.77, while *models B* and *C* predict values of 0.81 and 0.91 for the same variable, respectively. In the first model, by trading housing frequently, individuals manage to keep a smoother profile of non-durable consumption. More liquid durable assets are effectively used as a means to self-insure against idiosyncratic risk.

How Often Do People Move? *Model A* predicts a considerable amount of volatility in housing investment, both at the macro level (the volatility of housing investment exceeds its data counterpart by a factor of two) and at the micro level: without adjustment costs, individuals change their housing position every model period.

¹⁶ Citing from Khan and Thomas (in the New Palgrave Dictionary of Economics, forthcoming): «THOMAS J.M. (2002) and KHAN A. and THOMAS J.M. (2003; 2007) show that the target capital(s) selected by firms facing non-convex costs exhibit changes an order of magnitude smaller in general equilibrium. Because large movements in target capital, and hence in aggregate investment demand, would imply intolerable consumption volatility for households (at least in the closed-economy settings examined in these studies), they do not occur in equilibrium. Instead, small changes in relative prices serve to discourage sharp changes in optimal capital, thereby preventing large synchronizations in firms' investment timing and leaving the aggregate series largely unaffected by the microeconomic lumpiness caused by non-convex adjustment costs».

Model B requires households to pay the adjustment cost for every adjustment of the stock larger than 3 percent. With an adjustment cost equivalent to 3 percent of the housing stock (roughly 5 percent of annual labour income), individuals are reluctant to change their housing stock too much too quickly: housing is illiquid, but not very volatile. On average, individuals pay the adjustment cost for large changes only rarely. If paying the adjustment cost corresponds to moving, they do so once every 51 years. As a fraction of total residential investment, aggregate transaction costs ($\Omega(H_t, H_{t-1})$ in equation 1) — the model counterpart to the empirical brokers' commissions — are slightly less than 1 percent, as opposed to 10/15 percent in the data, as we reviewed in Section 2.

Model C (the pure lumpy model) predicts that people “move” every 13 years. This number is slightly lower than in the data (in the US, people move on average every 5 to 7 years), but our model abstracts from life-cycle effects, divorce and job reallocation, which can all cause people to move. Along this strictly microeconomic dimension, *model C* is the one that fits the data best. *Model C* also fits the data well in that it predicts a non-negligible share of residential investment that can be counted as transaction costs: aggregate transaction costs amount in fact to 4 percent of residential investment, although they are slightly countercyclical; the model predicts that people are more likely to move in a recession, since binding borrowing constraints force them to liquidate their home to keep servicing their debt: this is slightly at odds with the data, that typically show that housing sales and transactions are procyclical.

Cyclical Properties of Housing Investment. All the models replicate the positive correlation between housing investment and aggregate economic activity, although the correlation varies substantially across models.

The model without adjustment costs (*model A*) is the one that features the smallest correlation between housing investment and GDP: we also note that this model delivers very similar properties to its representative agent counterpart, where we assume two types of agents as well as borrowing constraints that are always binding for the impatient agents, always non-binding for the patient. In

model A, the correlation between housing investment and GDP is 0.31, and is substantially lower than its data counterpart of 0.64. Another property of the model without housing adjustment costs is that it delivers a negative correlation between household and business investment, with a value of -0.46 . This result stems from the fact that housing and non-housing capital are very close substitutes from the household perspective, and is a well known result in the household production literature: see for instance the discussion in Fisher (2007). Another characteristic of this model is that housing investment lags the business cycle, unlike in the data: intuitively, this happens because individuals prefer to accumulate non-housing capital faster and earlier when productivity improves, and build up homes only later.¹⁷

The model with a 3% band for the adjustment cost (*Model B*) features more procyclical residential investment. The contemporaneous correlation between housing investment and GDP is 0.61, although housing still lags GDP. However, the model matches well the positive correlation between housing and business investment, a value of 0.31 (in the data that we use for Table 1, this correlation — not reported in the Table — is 0.27).

A similar pattern emerges in the pure lumpy model: the contemporaneous correlation between housing and business investment is 0.50, although — again — housing investment tends to lag GDP (there is a higher correlation between current GDP and future housing investment than vice versa).

7. - Conclusions

In this paper we have posed a simple question: how does the microeconomic lumpiness of housing adjustment affect the equilibrium dynamic properties of aggregate consumption and investment? Our main conclusion is that lumpiness matters: in particular, we have shown that lumpiness in housing adjustment

¹⁷ See again FISHER J. (2007) for a recent discussion of these issues in the context of a representative agent model.

(1) reduces the volatility of both household and business investment; (2) increases the volatility of aggregate consumption; (3) increases the correlation of housing investment with business investment and with GDP. We have also noted that lumpiness of investment activity at the household level has significant aggregate implications, in contrast with the literature that shows that the aggregate effects of lumpy investment at the firm level are negligible.

APPENDIX A**Computational Details**

We numerically solve for the model equilibrium using a computational method similar to the one used in Krusell and Smith (1998). The value and policy functions are computed on grids of points for the state variables, and then approximated with linear interpolation at points not on the grids. The algorithm consists of the following steps:

1. Specify grids for the state space of individual and aggregate state variables.

As a result of robustness checks, the number of grid points was chosen as follows: 5 points for the aggregate shock, 3 values for the idiosyncratic shock, 60 points for the housing stock, and 550 points for the financial asset. For aggregate variables, we choose a grid of respectively 7 and 3 equally spaced points in the range $[0.9K^*, 1.1K^*]$ and $[0.9H^*, 1.1H^*]$, where K^* and H^* denote the steady state aggregate capital and housing stock respectively.

2. Guess initial coefficients $\{w_{is}^A\}_{A \in \tilde{A}, i=0, \dots, 2, s \in \{K, H, L\}}$ for the linear functions that approximate the laws of motion of aggregate capital, durables and labour:

$$K_t = w_{0K}^A + w_{1K}^A K_{t-1} + w_{2K}^A H_{t-1}$$

$$H_t = w_{0H}^A + w_{1H}^A K_{t-1} + w_{2H}^A H_{t-1}$$

$$L_t = w_{0L}^A + w_{1L}^A K_{t-1} + w_{2L}^A H_{t-1}$$

3. Use value function iteration in combination with Howard algorithm to compute optimal policies as a function of the individual and aggregate states.¹⁸ Notice that the intra-temporal

¹⁸ In computation, we exploit the strict concavity of the value function in the choice for assets as well as the monotonicity of the policy function in assets (given any choice for the durable good).

optimal value for labour hours as a function of consumption and productivity shock is the following:

$$l_t = \bar{l} - \frac{\tau c_t}{w_t z_t}$$

which allows one to derive consumption directly from the budget constraint as follows:

$$c_t = \frac{w_t z_t \bar{l} - R_t b_{t-1} + b_t + (1 - \delta_H) h_{t-1} - h_t - \Psi(h_t, h_{t-1})}{1 + \tau}$$

and to write the per-period utility function as

$$u = (1 + \tau) \log c_t + j \log h_t + \tau \log \tau - \tau \log(w_t z_t)$$

As a consequence, the individual dynamic optimization problem entails solving for policy functions for b and h only.

For $l_t = 0$, we have

$$\begin{aligned} & \max_{b_t, h_t} \{ \log c_t + j \log h_t + \beta_t E_t V(x_{t+1}; \beta_t) \} \\ \text{s.t. } & c_t = h_{t-1} (1 - \delta_H) - h_t - \Psi(h_t, h_{t-1}) - R_t b_{t-1} + b_t \end{aligned}$$

In practice, we prevent individuals from choosing zero hours.

4. Draw a series of aggregate and idiosyncratic shocks, and use the (approximated) policy functions and the predicted aggregate variables to simulate the optimal decisions of a large number of agents for many periods. We simulate 12,000 individuals for 10,000 periods, discarding the first 200.¹⁹ Compute the aggregate variables K_t , H_t and L at each t .

¹⁹ We “enforced” the law of large numbers by making sure that the simulated fractions of labour productivity shocks corresponded to the theoretical ones, by randomly adjusting the values of the shocks.

5. Run a regression of the simulated aggregate capital, durables and labour on past aggregate variables K and H , retrieving the new coefficients $\{w_{is}^A\}$ for the law of motion for H_t , L_t and K_t . We repeat steps 3 and 4 until convergence over the coefficients of the regressions. We measure goodness of fit using the R -squared of the regressions (which are always equal to 0.999 or higher at convergence).

APPENDIX B**Data Construction**

— Consumption (*C*): Real Personal Consumption Expenditure.

Source: Bureau of Economic Analysis (BEA). Available at:

<http://research.stlouisfed.org/fred2/data/PCECC96.txt>

— Residential Investment (*IH*): Real Private Residential Fixed Investment. Source: BEA. Available at:

<http://research.stlouisfed.org/fred2/data/PRFIC96.txt>

— Business Investment (*IK*): Real Private Non-residential Fixed Investment. Source: BEA. Available at:

<http://research.stlouisfed.org/fred2/data/PNFIC96.txt>

— *GDP*: Defined as $C + IH + IK$

Household debt (*DEBT*): Households (end of period, outstanding) home mortgage debt deflated with the BLS implicit price deflator for the non-farm business sector. Data are from the Flow of Funds Z1 release. The series code for mortgage debt is LA153165105.

The deflator can be found at <http://research.stlouisfed.org/fred2/data/IPDNBS.txt>

All data series are quarterly and detrended with an HP filter with $\lambda = 1600$.

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