Appendix (Not for Publication)

This Appendix presents details on the solution of agents' problems and the market clearing conditions. A separate Appendix detailing the derivation of the steady state is available upon request.

Agents' Decisions

Bankers

Bankers maximize their expected lifetime utility

$$\sum_{t=0}^{\infty} \beta_B^t \log C_{B,t}$$

subject to

$$C_{B,t} + R_{D,t}D_{t-1} + L_t + B_{B,t} + B_{F,t} =$$
$$D_t + R_{L,t}L_{t-1} + R_{B,t}B_{B,t-1} + R^*_{B,t}B_{F,t-1}$$
$$D_t \leq \gamma_L L_t + B_{B,t} + B_{F,t}.$$

Let $\mu_{B,t}$ be the Lagrange multiplier on the budget constraint and $\lambda_{B,t}$ be the lagrange multiplier on the capital requirement. Bankers choose C, D, L, B_B, B_F to get

$$\mu_{B,t} = C_{B,t}^{-1},$$
$$\mu_{B,t} - \beta_B \mu_{B,t+1} R_{D,t+1} - \lambda_{B,t} = 0,$$
$$-\mu_{B,t} + \beta_B \mu_{B,t+1} R_{L,t+1} + \lambda_{B,t}\gamma_L = 0,$$
$$-\mu_{B,t} + \beta_B \mu_{B,t+1} R_{B,t+1} + \lambda_{B,t} = 0,$$
$$-\mu_{B,t} + \beta_B \mu_{B,t+1} R^*_{B,t+1} + \lambda_{B,t} = 0.$$

Next, use a change in variables. Let $\tilde{\lambda}_{B,t} = \frac{\lambda_{B,t}}{\mu_{B,t}}$ or $\lambda_{B,t} = \mu_{B,t}\tilde{\lambda}_{B,t}$. Then, we can rewrite the conditions above

$$\mu_{B,t} = C_{B,t}^{-1},$$
$$\left(1 - \tilde{\lambda}_{B,t}\right)\mu_{B,t} = \beta_B \mu_{B,t+1} R_{D,t+1},$$
$$\left(1 - \tilde{\lambda}_{B,t}\gamma_L\right)\mu_{B,t} = \beta_B \mu_{B,t+1} R_{L,t+1},$$
$$\left(1 - \tilde{\lambda}_{B,t}\right)\mu_{B,t} = \beta_B \mu_{B,t+1} R_{B,t+1},$$
$$\left(1 - \tilde{\lambda}_{B,t}\right)\mu_{B,t} = \beta_B \mu_{B,t+1} R^*_{B,t+1}.$$

Let us express $R_{L,t+1}$ as a function of $R_{B,t+1}$. From above

$$\beta_B \mu_{B,t+1} R_{L,t+1} = \left(1 - \tilde{\lambda}_{B,t}\gamma_L\right)\mu_{B,t}, \tag{A1}$$

$$\beta_B \mu_{B,t+1} R_{B,t+1} = \left(1 - \tilde{\lambda}_{B,t}\right) \mu_{B,t}. \tag{A2}$$

Next, divide equation A1 by equation A2 and rearrange to obtain:

$$\mu_{B,t+1} R_{L,t+1} = \mu_{B,t+1} R_{B,t+1} \frac{\left(1 - \tilde{\lambda}_{B,t} \gamma_L\right)}{\left(1 - \tilde{\lambda}_{B,t}\right)}.$$

Similarly, using

$$\left(1 - \tilde{\lambda}_{B,t}\right) \mu_{B,t} = \beta_B \mu_{B,t+1} R^*_{B,t+1},$$
$$\left(1 - \tilde{\lambda}_{B,t}\right) \mu_{B,t} = \beta_B \mu_{B,t+1} R_{B,t+1},$$

one obtains

$$\mu_{B,t+1} R^*_{B,t+1} = \mu_{B,t+1} R_{B,t+1} \frac{\left(1 - \tilde{\lambda}_{B,t}\right)}{\left(1 - \tilde{\lambda}_{B,t}\right)}.$$

To summarize, the conditions for an equilibrium from the bankers' problem are

$$\begin{aligned} D_t &\leq \gamma_L L_t + \gamma_B B_{B,t} + \gamma_F B_{F,t}, \\ \mu_{B,t+1} R_{L,t+1} &= \mu_{B,t+1} R_{B,t+1} \frac{\left(1 - \tilde{\lambda}_{B,t} \gamma_L\right)}{\left(1 - \tilde{\lambda}_{B,t}\right)}, \\ \mu_{B,t+1} R^*_{B,t+1} &= \mu_{B,t+1} R_{B,t+1} \frac{\left(1 - \tilde{\lambda}_{B,t}\right)}{\left(1 - \tilde{\lambda}_{B,t}\right)}. \end{aligned}$$

Entrepreneurs

Entrepreneurs transform loans into capital using a one-for-one technology and can convert loans back into consumption. They maximize their expected lifetime utility

$$\sum_{t=0}^{\infty} \beta_E^t \log C_{E,t},$$

subject to

$$C_{E,t} + K_t + R_{L,t} L_{t-1} = R_{K,t} K_{t-1} + (1-\delta) K_{t-1} + L_t,$$
$$L_t = \rho_E L_{t-1} + (1 - \rho_E) m K_{Et}.$$

Introduce a change in variables. Let $\tilde{\lambda}_{E,t} = \frac{\lambda_{E,t}}{\mu_{E,t}}$ or $\lambda_{E,t} = \mu_{E,t} \tilde{\lambda}_{E,t}$. Then, the optimizing conditions can be written as, using $\mu_{E,t} = C_{E,t}^{-1}$:

$$\left(1 - \tilde{\lambda}_{E,t}\right) \mu_{E,t} = \beta_E \left(R_{L,t+1} - \rho_E \tilde{\lambda}_{E,t+1}\right) \mu_{E,t+1}$$
$$\left(1 - \tilde{\lambda}_{E,t} (1 - \rho_E) m\right) \mu_{E,t} = \beta_E (R_{K,t+1} + 1 - \delta) \mu_{E,t+1}$$

Government

The government is assumed be myopic and have utility given by $u_t = v(G_t)$ (any function increasing in G_t would do, given the simple nature of the problem). Under this setup, the government default if it can achieve greater spending today by defaulting relative to the no default case. Given the debt ceiling constraint of the government, default can take place if a reduction in GDP tightens the borrowing constraint so much that the government finds it convenient to repudiate its debt rather than to pay back its previous obligations.