Macro Topics

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May 2013

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Course overview

We will discuss two broad topics

- Solving DSGE models, with particular attention to their nonlinearities
- Formulating (and estimating) DSGE models with financial frictions, with particular attention to debt and housing

DSGE Models: The Simplest Example

The planner's problem can be written as:

$$\max E_t \left(\sum_{s=t}^{\infty} \beta^{s-t} \log C_s \right)$$

subject to

$$K_t - K_{t-1} = A_t^{1-\alpha} K_{t-1}^{\alpha} - C_t \tag{1}$$

Optimal consumption implies

$$\frac{1}{C_t} = \beta E_t \left(\frac{1}{C_{t+1}} \left(\alpha \left(\frac{A_{t+1}}{K_t} \right)^{1-\alpha} + 1 \right) \right) \tag{2}$$

Assume technology follows an AR(1) process in logs, that is

$$\log A_t = \rho \log A_{t-1} + \log U_t \tag{3}$$

where ρ is the autocorrelation of the shock. Assume that $\log U$ has mean zero, finite variance.

The system made by (1) to (3) is a non-linear system with rational expectations. We usually solve them in the following steps

- Find the steady state.

$$
\begin{aligned}
\log A &= 0- > A = 1 \\
C &= K^{\alpha} \\
1-\beta &= \alpha\beta\left(\frac{1}{K}\right)^{1-\alpha} - > \left(\frac{\alpha\beta}{1-\beta}\right)^{\frac{1}{1-\alpha}} = K
\end{aligned}
$$

- Linearize around the steady state

equation 1

$$C_t = A_t^{1-\alpha} K_{t-1}^{\alpha} - K_t + K_{t-1}$$

$$\begin{aligned}
\log C_t &= \log \left(A_t^{1-\alpha} K_{t-1}^{\alpha} - K_t + K_{t-1} \right) \\
&\quad \text{take total differential around steady state} \\
\frac{1}{C} dC_t &= \frac{1}{C} \left((1-\alpha) A^{-\alpha} K^{\alpha} dA_t + \alpha A^{1-\alpha} K^{\alpha-1} dK_{t-1} - dK_t + dK_{t-1} \right) \\
c_t &= (1-\alpha) A_t + \alpha k_{t-1} - \frac{K}{C} k_t + \frac{K}{C} k_{t-1}
\end{aligned}$$

- equation 2

$$
\begin{aligned}
E_t\left(\frac{C_{t+1}}{C_t}\right) &= \beta E_t\left(\alpha\left(\frac{A_{t+1}}{K_t}\right)^{1-\alpha}+1\right)\\
&\text{steady state of both sides is 1}\\
E_t c_{t+1}-c_t &= \alpha\left(1-\alpha\right)\left(\frac{1}{K}\right)^{1-\alpha}\left(E_t a_{t+1}-k_t\right)\\
0 &= -E_t c_{t+1}+c_t+\frac{\left(1-\alpha\right)\left(1-\beta\right)}{\beta}\left(E_t a_{t+1}-k_t\right)
\end{aligned}
$$

- equation 3

$$a_t=\rho a_{t-1}+u_t$$

Taking Stock

This is a dynamic system of 3 equations in 3 unknowns. To use a more compact notation, we prefer to write it in the following form

$$0 = E_t\left[\mathbf{F}\mathbf{x}_{t+1} + \mathbf{G}\mathbf{x}_t + \mathbf{H}\mathbf{x}_{t-1} + \mathbf{L}\mathbf{z}_{t+1} + \mathbf{M}\mathbf{z}_t\right] \quad (4)$$

$$\mathbf{z}_t = \mathbf{N}\mathbf{z}_{t-1} + \mathbf{e}_t \quad (5)$$

where

- $\mathbf{x}_t$ is the vector collecting all the endogenous variables of the model.
- $\mathbf{z}_t$ collects all the exogenous stochastic processes.

In our above example

$$\mathbf{x} = \begin{bmatrix} c \\ k \end{bmatrix}, \mathbf{z} = [a]$$

and $\mathbf{F} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} -1 & -\frac{\alpha\beta}{1-\beta} \\ 1 & -\frac{(1-\alpha)(1-\beta)}{\beta} \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 0 & \frac{\alpha}{1-\beta} \\ 0 & 0 \end{bmatrix},$

$\mathbf{L} = \begin{bmatrix} 0 \\ \frac{(1-\alpha)(1-\beta)}{\beta} \end{bmatrix}, \mathbf{M} = \begin{bmatrix} 1-\alpha \\ 0 \end{bmatrix}, \mathbf{N} = [\rho]$

To summarize, a linearized DSGE model can be written in the following form

$$
\begin{aligned}
0 &= E_t \left[\mathbf{F}\mathbf{x}_{t+1} + \mathbf{G}\mathbf{x}_t + \mathbf{H}\mathbf{x}_{t-1} + \mathbf{L}\mathbf{u}_{t+1} + \mathbf{M}\mathbf{u}_t \right] \\
\mathbf{z}_t &= \mathbf{N}\mathbf{z}_{t-1} + \mathbf{e}_t
\end{aligned}
$$

The recursive equilibrium law of motion describes endogenous variables as function of the STATE:

$$
\mathbf{x}_t = \mathbf{P}\mathbf{x}_{t-1} + \mathbf{Q}\mathbf{z}_t \tag{6}
$$

i.e., matrices **P**, **Q** such that the equilibrium is described by these rules.

- Finally, what we do is to plug the matrices in (4) and (5) in a computer, to obtain (6) .

In our toy example above, set $\alpha = 0.33, \beta = 0.99, \rho = 0.98$. Then

$$
\begin{bmatrix} c_t \\ k_t \end{bmatrix} = \begin{bmatrix} 0 & 0.6589 \\ 0 & 0.9899 \end{bmatrix} \begin{bmatrix} c_{t-1} \\ k_{t-1} \end{bmatrix} + \begin{bmatrix} 0.1755 \\ 0.0151 \end{bmatrix} [z_t]
$$

Nonlinear models

- Nonlinear approximations of structural models can be represented using three equations, written with variables typically expressed in levels.
- The first equation is

$$s_t = f\left(s_{t-1}, v_t\right)$$

  where s_t collects the state variables, v_t the collection of structural shocks incorporated in the model.
- The second equation is the policy function, which incorporates the optimal specification of the control variables c_t included in the model as a function of the shocks

$$c_t = c\left(s_t\right)$$

- The third equation maps the full collection of model variables into the observables

$$X_t = \widetilde{g}\left(s_t, c_t, v_t, u_t\right)$$

  where the presence of u_t reflects the possibility that the observations of X_t are associated with measurement error.

Occbin: Toolkit to Solve Models with Occasionally Binding Constraints

Occasionally binding constraints arise in many economic applications.
Examples include:
Models with limitations on the mobility of factors of production;

Models with heterogenous agents and constraints on the financial assets available to agents;

Models with a zero lower bound on the nominal interest rate;

Models with inventory management.

Why is a Toolkit Needed?

Encompassing realistic features to improve model fit in empirically driven applications may quickly raise the number of state variables. This may render standard global solution methods, such as dynamic programming, infeasible.

An alternative that has been used in practice, especially in applications that deal with the zero lower bound on policy rates, is to use a piece-wise perturbation approach.

This approach has the distinct advantage of delivering a solution for models with a large number of state variables. Furthermore, it can be easily extended to encompass multiple occasionally binding constraints.

Related Stuff

Jung, Teranishi, and Watanabe introduced this approach (JMCB, 2005). The approach has been adopted, among others, by Eggertson and Woodford (Carnegie Rochester, 2003), and by Christiano, Eichenbaum, and Rebelo (JPE, 2011).
Other methods have been used and proposed.

1) Show a toolbox that extends Dynare to use this solution technique.
2) For simple models, we gauge the performance of the piece-wise perturbation approach relative to the dynamic programming solution.

The Solution Method

- The linearized system of necessary conditions for an equilibrium of a baseline DSGE model can be expressed as:

$$\mathcal{A}_1 E_t X_{t+1} + \mathcal{A}_0 X_t + \mathcal{A}_{-1} X_{t-1} + \mathcal{B} u_t = 0. \qquad \text{(M1)}$$

  There are situations however when one of the conditions describing the equilibrium does not hold, and is replaced by another one.When the "starred" system applies, the linearized system can be expressed as:

$$\mathcal{A}_1^* E_t X_{t+1} + \mathcal{A}_0^* X_t + \mathcal{A}_{-1}^* X_{t-1} + \mathcal{B}^* u_t + \mathcal{C}^* = 0 \qquad \text{(M2)}$$

  where C^* is a vector of constants.
- Both systems are linearized around the same point – same X across systems.

The Solution Method

- When the baseline model applies (M1), we use standard methods to express solution as:

$$X_t = \mathcal{P} X_{t-1} + \mathcal{Q} u_t. \qquad \text{(M1\_DR)}$$

- If the starred model applies (M2), shoot back towards the initial condition from the first period when constraint binds again.
- Main idea: suppose that M1 applied in $t-1$, M2 applies in t, but M1 is expected to apply in all future periods $t+1$, the decision rule in t is:

$$\underbrace{\mathcal{A}_1^*}_{\text{M2}} \underbrace{\mathcal{P} X_t}_{\text{M1\_DR}} + \mathcal{A}_0^* X_t + \underbrace{\mathcal{A}_{-1}^*}_{\text{M2}} X_{t-1} + \mathcal{C}^* = 0,$$

$$X_t = -\left(\mathcal{A}_1^* \mathcal{P} + \mathcal{A}_0^*\right)^{-1} \left(\mathcal{A}_{-1}^* X_{t-1} + \mathcal{B}^* u_t + \mathcal{C}^*\right)$$

- One can proceed in a similar fashion to construct the time-varying decision rules when M2 applies for multiple periods.
- In each period in which M2 applies, the expectation of how long one expects to stay in M2 affects the value of X_t today

The Solution Method

The search for the appropriate time-varying decision rules implies that for each set of shocks at a point in time one needs to calculate the expected future duration of each "regime."

Truncate the simulation at an arbitrary point and reject the truncation if the solution implies that the model has not returned to the reference regime by that point.

Start with a guess of the expected durations that is based on the linear solution. Update the guess based on where the conditions of system 1 are violated using the piece-wise linear method until no violation remains.

An Example

- To fix ideas, let's first consider a simple, forward-looking, linear model:

$$
\begin{aligned}
q_t &= \beta(1-\rho)E_t q_{t+1} + \rho q_{t-1} - \sigma r_t + u_t \\
r_t &= \max\left(\underline{r}, \phi q_t\right)
\end{aligned}
$$

  where u_t is an *iid* shock.

- The general solution for q_t takes the form

$$
\begin{aligned}
q_t &= \varepsilon_{qq,t} q_{t-1} + \varepsilon_{qu,t} u_t + c_{q,t} \\
r_t &= \varepsilon_{rr,t} q_{t-1} + \varepsilon_{ru,t} u_t + c_{r,t}
\end{aligned}
$$

- In turn, the ε are functions of q_{t-1} and u_t.
- How do we find the solution?

An Example: Solution ignoring the constraint

Ignore the constraint first

$$\begin{aligned} q_t &= \frac{\beta(1-\rho)}{1+\sigma\phi} E_t q_{t+1} + \frac{\rho}{1+\sigma\phi} q_{t-1} + \frac{1}{1+\sigma\phi} u_t \\ q_t &= a E_t q_{t+1} + b q_{t-1} + c u_t \end{aligned}$$

Find solution (method of undetermined coefficients)

$$\begin{aligned} q_t &= \varepsilon_q q_{t-1} + \varepsilon_u u_t \text{ (guess)} \\ E_t q_{t+1} &= \varepsilon_q q_t \text{ (expectation given guess)} \\ a E_t q_{t+1} &= a\varepsilon_q q_t = a\varepsilon_q^2 q_{t-1} + a\varepsilon_q \varepsilon_u u_t \end{aligned}$$

Match coefficients

$$\underbrace{e q_{t-1} + \varepsilon_u u_t}_{q_t} = \underbrace{a\varepsilon_q^2 q_{t-1} + a\varepsilon_q \varepsilon_u u_t}_{aE_t q_{t+1}} + b q_{t-1} + c u_t$$

so that

$$\varepsilon_q = a\varepsilon_q^2 + b, \ \varepsilon_u = a\varepsilon_q \varepsilon_u + c$$

$$\varepsilon_q = \left(1 - \sqrt{1-4ab}\right)/2a, \varepsilon_u = c/\left(1 - a\varepsilon_q\right)$$

Plug some numbers

$$\beta = 0.99$$
$$\phi = 1$$
$$\rho = 0.5$$
$$\sigma = 1$$
$$\underline{r} = -0.02$$

In this case

$$\varepsilon_q = 0.2677$$
$$\varepsilon_u = 0.5355$$

so that

$$q_t = r_t = 0.2677 q_{t-1} + 0.5355 u_t$$

Is this solution always correct? Consider the case of a large negative shock to u. If $q_{t-1} = 0$, any u_t such that

$$\begin{aligned} r^* &= 0.5355u^* < -0.02 \\ u^* &< -0.0373 \end{aligned}$$

will violate constraint.
Suppose for instance $u_1 = -0.2$. Ignoring constraint, solution is

$$r_t = 0.2677r_{t-1} + 0.5355u_t$$

$$\begin{aligned} r_1 &= -0.5355 * 0.2 = -0.1071 \\ r_2 &= 0.2677r_1 = -0.0287 \\ r_3 &> -0.02 \end{aligned}$$

Hence ignoring the constraint r_t would be below -0.02 for 2 periods. Moreover, there is a feedback loop. Higher values of r imply lower q, which implies lower desired values of r, so r can end up being at $\underline{r}$ for longer

We use a guess and verify method to determine how long the constraint will bind. We start by guessing durations that are based on the linear solution that ignores the constraint. Iterate until convergence.

So the first guess is going to be 2 periods.

–> Suppose we guess that r remains at ϕ for $t\_low = 2$ periods.
–> Because r is not going to be low as guessed in linear solution ....
–> q will fall more than if r did not hit the constraint ...
–> and r might in turn stay at its lowest bound ϕ more than t_low periods.

In all interesting cases, first guess is not last guess, since dynamics of system depend on feedback loop between duration of constraint and endogenous reaction of variables to constraint. In the example above, one can think of a New Keynesian model at the ZLB.

Now cast system using our general notation (use $\beta' = \beta(1-\rho)$):

$$\begin{aligned} q_t &= \beta' E_t q_{t+1} + \rho q_{t-1} - \sigma r_t + u_t \\ r_t &= \max(\underline{r}, \phi q_t) \end{aligned}$$

$$\mathcal{A}_1 E_t X_{t+1} + \mathcal{A}_0 X_t + \mathcal{A}_{-1} X_{t-1} + \mathcal{B} u_t = 0. \qquad \text{(M1)}$$

$$\begin{bmatrix} -\beta' & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ r_1 \end{bmatrix} + \begin{bmatrix} 1 & \sigma \\ -\phi & 1 \end{bmatrix} \begin{bmatrix} q \\ r \end{bmatrix} + \begin{bmatrix} -\rho & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} ql \\ rl \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} u = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and

$$\mathcal{A}_1^* E_t X_{t+1} + \mathcal{A}_0^* X_t + \mathcal{A}_{-1}^* X_{t-1} + \mathcal{B}^* u_t = -\mathcal{C}^* \qquad \text{(M2)}$$

$$\begin{bmatrix} -\beta' & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ r_1 \end{bmatrix} + \begin{bmatrix} 1 & \sigma \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q \\ r \end{bmatrix} + \begin{bmatrix} -\rho & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} ql \\ rl \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} u = \begin{bmatrix} 0 \\ \bar{r} \end{bmatrix}$$

and

$$X_t = \mathcal{P} X_{t-1} + \mathcal{Q} u_t \qquad \text{(M1\_DR)}$$

$$\begin{bmatrix} q_t \\ r_t \end{bmatrix} = \begin{bmatrix} \varepsilon_q & 0 \\ \varepsilon_q & 0 \end{bmatrix} \begin{bmatrix} q_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_u \\ \varepsilon_u \end{bmatrix} u$$

We guess that in period 3 normal system applies. In that case, the solution in period 2 should satisfy

$$
\begin{aligned}
X_2 &= -\left(\mathcal{A}_1^*\mathcal{P} + \mathcal{A}_0^*\right)^{-1}\left(\mathcal{A}_{-1}^* X_1 + \mathcal{B}^* u_2 + \mathcal{C}^*\right) \\
&= P_2 X_1 + Q_2 u_2 + C_2
\end{aligned}
$$

where

$$
\begin{aligned}
P_2 &= -\left(\mathcal{A}_1^*\mathcal{P} + \mathcal{A}_0^*\right)^{-1}\mathcal{A}_{-1}^*, \\
Q_2 &= -\left(\mathcal{A}_1^*\mathcal{P} + \mathcal{A}_0^*\right)^{-1}\mathcal{B}^* u_2, \quad C_2 = -\left(\mathcal{A}_1^*\mathcal{P} + \mathcal{A}_0^*\right)^{-1} C^*
\end{aligned}
$$

I plug the numbers now. Using

$$
\begin{aligned}
\mathcal{A}_1^* &= \begin{bmatrix} -0.495 & 0 \\ 0 & 0 \end{bmatrix}, \mathcal{P} = \begin{bmatrix} 0.2677 & 0 \\ 0.2677 & 0 \end{bmatrix}, \mathcal{A}_0^* = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\
\mathcal{A}_{-1}^* &= \begin{bmatrix} -0.495 & 0 \\ 0 & 0 \end{bmatrix}, \mathcal{B}^* = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \mathcal{C}^* = \begin{bmatrix} 0 \\ 0.02 \end{bmatrix}
\end{aligned}
$$

I get

$$
X_2 = \underbrace{\begin{bmatrix} 0.5706 & 0 \\ 0 & 0 \end{bmatrix}}_{P_2} X_1 + \underbrace{\begin{bmatrix} .023 \\ -0.02 \end{bmatrix}}_{C_2}
$$

We do not know the solution in period 1. However, assuming the starred system applies in $t = 1$ and is expected to apply in $t = 2$, the solution in 1 is

$$\mathcal{A}_1^* \left(\mathcal{P}_2 X_1 + C_2\right) + \mathcal{A}_0^* X_1 + A_{-1}^* X_0 + B u_1 + \mathcal{C}^* = 0,$$
$$X_1 = -\left(\mathcal{A}_1^* \mathcal{P}_2 + \mathcal{A}_0^*\right)^{-1} \left(\mathcal{B}^* u_1 + \mathcal{C}^* + A_1^* C_2 + A_{-1}^* X_0\right)$$
$$X_1 = P_1 X_0 + Q_1 u_1 + C_1$$

where

$$\begin{aligned} P_1 &= -\left(\mathcal{A}_1^* \mathcal{P}_2 + \mathcal{A}_0^*\right)^{-1} A_{-1}^* \\ Q_1 &= -\left(\mathcal{A}_1^* \mathcal{P}_2 + \mathcal{A}_0^*\right)^{-1} \mathcal{B}^* \\ C_1 &= -\left(\mathcal{A}_1^* \mathcal{P}_2 + \mathcal{A}_0^*\right)^{-1} \left(\mathcal{C}^* + A_1^* C_2\right) \end{aligned}$$

After plugging in all the numbers, assuming $X_0 = 0$, we get

$$X_1 = \begin{bmatrix} -0.235\,00 \\ -0.02 \end{bmatrix}$$

Now I plug X_1 back and get

$$X_2 = \begin{bmatrix} 0.57064 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -0.23500 \\ -0.02 \end{bmatrix} + \begin{bmatrix} .023 \\ -0.02 \end{bmatrix} = \begin{bmatrix} -0.1111 \\ -0.02 \end{bmatrix}$$

and now I plug X_2 into X_3

$$\begin{aligned} X_3 &= \mathcal{P} X_2 \\ X_3 &= \begin{bmatrix} -0.0297 \\ -0.0297 \end{bmatrix} \end{aligned}$$

which violates constraint in 3.

- Note the need to update guess.
- We guessed that the starred system applies in 2 and that the normal applies in 3. Based on this guess, the starred system applies in 3.
- Hence we update the guess that starred system applies for 3 periods.
- Redo the whole thing again until the guessed duration in the starred regime coincides with the actual duration.

In the next step, we assume that the normal system applies in 4 but the starred applies in 1, 2 and 3, solve for P_3, Q_3 and C_3, use them to compute X_2 and X_1, and go back to see if X_3 satisfies the constraints. (it does, I have checked it myself)

Example 1: Borrowing Constraint Model

- To check if method works, we apply it to models for which we can compute a a full non-linear solution to arbitrary precision using dynamic programming methods.
- Consider simple model where a random endowment y_t that can be used as collateral

$$u = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \log (c_t) \right\}$$

$$c_t = y_t + b_t - 1.05 b_{t-1}$$

$$b_t \leq 2y_t \tag{c1}$$

$$\log (y_t) = \rho \log (y_{t-1}) + \sigma \sqrt{1-\rho^2} \varepsilon_t$$

$$\varepsilon_t \tilde{} N(0,1), \ \sigma = 0.03, \ \rho = 0.9$$

- We look at how solution method handles cases when increases in y_t are large enough so that constraint is not binding. We try $\beta = 0.94$ and $\beta = 0.949$
- Here: constraint $(c1)$ BINDS in *normal* times.

Example 2: RBC with Irreversible Capital

- Investment cannot fall below a given threshold

$$
\begin{aligned}
u = E_0 \left\{ \textstyle\sum_{t=0}^{\infty} 0.96^t \log\left(c_t\right) \right\} \\
c_t + k_t - 0.9 k_{t-1} = A_t k_{t-1}^{0.33} \\
k_t - 0.9 k_{t-1} \geq \phi k_{t-1} \qquad \text{(c2)} \\
\log\left(A_t\right) = 0.9 \log\left(A_{t-1}\right) + \sigma \sqrt{1-\rho^2} \varepsilon_t \\
\varepsilon_t \tilde{} N\left(0,1\right),\ \sigma = 0.03,\ \rho = 0.9
\end{aligned}
$$

where $\phi > 0$.

- Here: constraint $(c2)$ DOES NOT bind in *normal* times.

Example 3: Borrowing and Housing

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t + j \log h_t \right)$$

$$\begin{aligned} c_t + q_t h_t &= y + b_t - R b_{t-1} + q_t h_{t-1} (1 - \delta) \\ b_t &\leq m q_t h_t \\ \log q_t &= \rho \log q_{t-1} + v_t \end{aligned}$$

Here the FOCs would be

$$\begin{aligned} \mu_t \left(b_t - m q_t h_t \right) &= 0 \\ u' (c_t) &= \beta R E_t u' (c_{t+1}) + \mu_t \end{aligned}$$

Assuming $\beta R < 1$, here the borrowing constraint binds in normal times.

Structure of Solution Programs (Dynare)

The programs we devised take as input two Dynare model files.
One .mod file specifies the normal M1 model from which we calculate

$$A_1 E_t X_{t+1} + A_0 X_t + A_{-1} X_{t-1} = 0.$$

The other .mod file specifies the starred model M2 with the occasionally binding constraint inverted (binding if it was not binding in the reference model, or not binding if it was binding in the reference model). This .mod file yields

$$A_1^* E_t X_{t+1} + A_0^* X_t + A_{-1}^* X_{t-1} + C^* = 0.$$

We use the analytical derivatives computed by Dynare to construct $A_1, A_0, A_{-1}, A_1^*, A_0^*, A_{-1}^*$, and C^*.

M1: hp.mod

y=1;
c+q*h=y+b-R*b(-1)+q*h(-1)*(1-δ);
b=M*q*h;
lb=1/c-β*R/c(+1);
q/c=j/h+β*(1-
δ)*q(+1)/c(+1)+lb*M*q;
lev=b/(M*q*h)-1;
log(q)=ρ*log(q(-1))+u;

M2: hpnotbinding.mod

y=1;
c+q*h=y+b-R*b(-1)+q*h(-1)*(1-δ);
lb=0;
lb=1/c-β*R/c(+1));
q/c=j/h+β*(1-
δ)*q(+1)/c(+1)+lb*M*q;
lev=b/(M*q*h)-1;
log(q)=ρ*log(q(-1))+u;

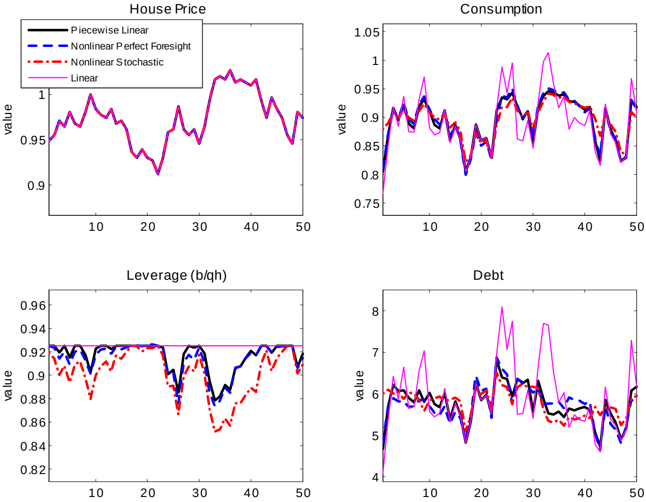
The main file `runsim_houseprice` contains

1. mod files: `model1 = 'hp'; model2 = 'hpnotbinding';`
2. constraint violation triggers switch to m2: `constraint='lb<-LBSS';`
3. constraint violation triggers switch to m1: `constraint_relax1='lev>0'`
4. solve model1 to obtain M1_DR; write model2 in M2 form
5. given shocks, check if model1 assumptions are violated, if so, look for solution

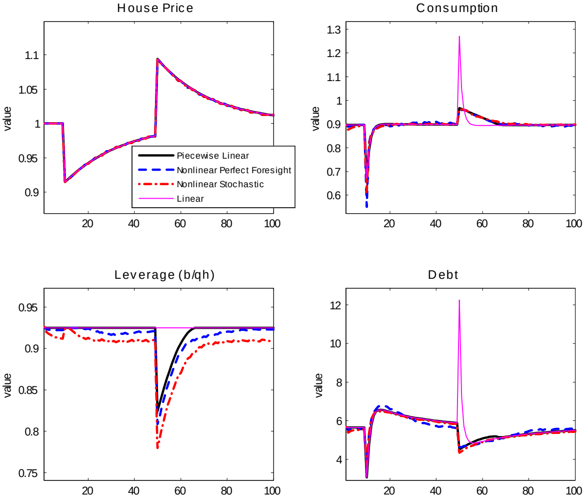
function [zdata zdataconcatenated oobase_ Mbase_] = ...
solve_one_constraint(model1,model2,...
constraint, constraint_relax,...
shockssequence,irfshock,nperiods,tol0,maxiter)

- `model1, model2`: dynare mod files containing (linear or nonlinear) model equations
- `constraint, constraint_relax`: strings with constraints that have to be verified
  constraint defines the first constraint
  if constraint is true, solution switches to model2
  but if constraint_relax is true, solution reverts to model1
- `shockssequence`: `sequence of innovations under which one wants to solve model`
  e.g. `randn(100,1)*`σ_j for simulation
  or `[ 1; zeros(50,1) ]` for impulse responses

Simulations - Borrowing and Housing Model



IRF - Borrowing and Housing Model



Accuracy – Borrowing and Housing Model

	Log Consumption		Correlations		$\frac{b}{qh}$	ΔWelf.
	st.dv	skewness	$\ln q, \ln c$	$\ln q, \frac{b}{qh}$	mean	
Linear	6.1%	0.03	0.40	0.00	0.925	0.18%
Occbin	4.5%	-1.17	0.54	-0.60	0.911	0.02%
Nonlin.perf.fores.	4.6%	-1.20	0.53	-0.58	0.910	0.01%
Nonlinear stoch.	3.7%	-1.30	0.65	-0.71	0.896	—

A DSGE Model with Multiple Financial Frictions

- Can financial frictions can explain of the quantitative effects of the financial crisis?
- Elements of the Financial crisis
  1. financial institutions suffer losses which impair their ability to extend credit to the real sector, causing a recession.
  2. borrowers balance sheets are impaired, causing a drop in spending
  3. credit supply is tight
- Here we take to the data a model which embeds these elements
- Model elements: banks and heterogeneous agents
  Event triggering cycles: (1) financial shocks; (2) changes in asset values; (3) changes in credit supply.

Main Findings

- Financial frictions and shocks in the financial sector account for more than half of the decline in GDP during the last recession
- Most promising shocks:
  Declines in asset values (2006-2007)
  Shocks hitting balance sheet of banks (2008-2009)
  Tightening of credit standards (2009-2010)
- We will see how to model them

Setup

1. **Households.**
   Some households are **Savers**: buy homes, supply deposits D to banking sector and do not face credit constraints.
   Some households are **Borrowers**: borrow L_S against their homes.
2. **Banks** collects deposits from savers and make loans for household borrowers and entrepreneurs.
3. **Entrepreneurs** borrow from bank, transforms L into K.
4. Competitive firm rents K and N to produce final good Y.
   HH savers and HH borrowers controlled by wage share in production.
   Size of Entrepreneurs controlled by capital share in production.
5. Shocks
   Borrowers are subject to exogenous **repayment shocks**: they may pay back less agreed
   Changes in **asset values** and **loan-to-values** affect ability to borrow and spend
   The usual suspects (TFP and preference)
   Hidden in the background, bells and whistles needed to take model

Household Savers

Choose consumption and deposits (savings) and hours worked

$$\max E_0 \sum_{t=0}^{\infty} \beta_H^t \left(\log C_{H,t} + j_t \log H_{H,t} + \tau \log \left(1 - N_{H,t} \right) \right)$$

s.t.

$$C_{H,t} + D_t + K_{H,t} + q_t \Delta H_{H,t} =$$
$$\left(R_{M,t} + 1 - \delta_K \right) K_{H,t-1} + R_{H,t-1} D_{t-1} + W_{H,t} N_{H,t}$$

If these households are not constrained, the equilibrium return on savings will settle around $1/\beta_H$ in steady state.

Household Borrowers

Low discount factor, creates simple motive for borrowing $\beta_S < \beta_B$ (s stands for subprime)

$$\max E_0 \sum_{t=0}^{\infty} \beta_S^t \left(\log C_{S,t} + j_t \log H_{S,t} + \tau \log \left(1 - N_{S,t} \right) \right)$$

s.t.

$$C_{S,t} + q_t \Delta H_{S,t} + R_{S,t-1} L_{S,t-1} - \varepsilon_t = L_{S,t} + W_{S,t} N_{S,t}$$
$$L_{S,t} \leq E_t \left(\frac{1}{R_{S,t}} m_{S,t} q_{t+1} H_{S,t} \right)$$

If β_S is low enough, the constraint on borrowing will hold in a neighborhood of the steady state.
ε_t is the repayment shock

Entrepreneurs

Entrepreneurs borrow L_E. Transform loan into capital using one-for-one technology, rent capital to representative firm.

$$\max E_0 \sum_{t=0}^{\infty} \beta_E^t \log C_{E,t}$$

where $\beta_E < \frac{1}{\gamma_E \frac{1}{\beta_H} + (1-\gamma_E)\frac{1}{\beta_B}}$ (β_B is banker's discount factor)
s.t.:

$$C_{E,t} + K_{E,t} + R_{E,t} L_{E,t-1} + q_t \Delta H_{E,t} = \\ L_{E,t} + (R_{K,t} + 1 - \delta) K_{E,t-1} + (R_{V,t} + 1) H_{E,t-1}$$

$$L_{E,t} \leq m_{E,t} K_{E,t} + m_H E_t q_{t+1} H_{E,t} / R_{E,t} - W_t N_t$$

Borrowing constraint binds if $\beta_E R_E < 1$ which occurs if R_E sufficiently low. (R_E tied to lenders' discount factor)

Bankers

1. Bankers transform savings into loans. To do so, they are required to hold some equity (bank capital) in their business
2. Bankers are shortsighted: blinded by greed/impatience, they try and borrow as much as they can from household to increase the size of their balance sheet.

Bankers

The banker's problem

$$\max E_0 \sum_{t=0}^{\infty} \beta_B^t \log C_{B,t}$$

where $\beta_B < \beta_H$, subject to:

$$C_{B,t} + R_{H,t-1} D_{t-1} + L_{E,t} + L_{S,t} = R_{E,t} L_{E,t-1} + R_{S,t-1} L_{S,t-1} + D_t - \varepsilon_t$$

[ε_t repayment shock]
and additional constraint:

$$D_t \leq \gamma \left(L_{E,t} + L_{S,t} - \varepsilon_t \right) \leftarrow \text{capital adequacy constraint (CAC)}$$

CAC forces banker to hold some equity in the business

Bank's optimality conditions for D, L_E (similar for L_S):

$$\begin{aligned} 1-\lambda_{B,t} &= E_t\left(m_{B,t}R_{H,t+1}\right) \\ 1-\gamma\lambda_{B,t} &= E_t\left(m_{B,t}R_{E,t+1}\right) \end{aligned}$$

- Expression for spread:

$$R_{E,t}-R_{H,t}=\frac{\lambda_{B,t}}{m_{B,t}}\left(1-\gamma_E\right).$$

  λ_B : multiplier of bank's capital constraint
  m_B : banker's stochastic discount factor

- Spread is larger when banker's constraint gets tigher (λ_B rises)
- When constraint gets tighter, bank requires larger compensation on loans to be indifferent b/w making loans and issuing deposits. Loans are more illiquid than deposits: when constraint is binding, a reduction in deposits of 1\$ requires cutting back on loans by $\frac{1}{\gamma_E}$\$.
- Rise in spread depresses activity when bank net worth is low.

Firms

Simple Static problem: rent capital from entrepreneurs and hire labor. Production function is:

$$Y_t = K_{E,t-1}^{\alpha\mu} K_{H,t-1}^{\alpha(1-\mu)} H_{E,t-1}^{\nu} N_{H,t}^{(1-\alpha-\nu)(1-\sigma)} N_{S,t}^{(1-\alpha-\nu)\sigma}$$

(variable capital utilization allowed for in estimation)

Steady State

$$R_H = \frac{1}{\beta_H} \leftarrow \text{return on HH savings}$$

$$\lambda_B = 1 - \beta_B R_H = 1 - \frac{\beta_B}{\beta_H} > 0 \text{ banker is constrained}$$

$$R_E - R_H = (1-\gamma)\left(\frac{1}{\beta_B} - \frac{1}{\beta_H}\right) > 0 \leftarrow \text{spread}$$

Hence $R_E > R_H$ (positive banking spreads):

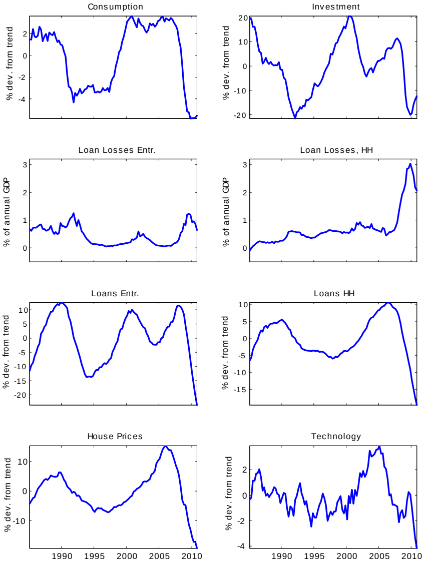
1. Return on bank loans must compensate banker for higher impatience
2. ... must be higher than cost of deposits to make up for higher "liquidity" of loans relative to deposits
   The larger γ, the more loans become substitutes with deposits in the capital adequacy constraint, the lower the extra return on loans required for the bank to be indifferent between borrowing and lending.

Calibration

1. Real return on saving 3% per year
2. Capital output ratio about 2.05, Housing output ratio 1.4
3. m_E, m_S, m_H: 90%, m_K: 50% –> Total debt to GDP 105%
4. For added quantitative realism:
   – inertia in the capital adequacy constraint
   $K_{B,t} > \rho_B K_{B,t-1} + (1-\rho_B)\left((1-\gamma)\left(L_{E,t} + L_{S,t} - \varepsilon\right)\right)$
   – quadratic capital adjustment costs

Estimation

1. Estimate (with Bayesian methods) μ (capital share of constrained entrepreneurs), ν (share of real estate for entrepreneurs) ϕ (adjustment costs), AR(1) process for loan losses ε_t, parameters describing inertia in borrowing and capital adequacy constraints, curvature utilization function, wage share of constrained HH σ.
2. Use data on
   - Consumption
   - Investment
   - Losses on Loans to Firms
   - Losses on Loans to Households
   - Loans to Households
   - Loans to Firms
   - Housing prices
   - TFP (utilization–adjusted measures from Fernald)
3. 8 shocks (housing demand j, financial shocks 1/2 b_e, b_h, LTV shocks m_e, m_h, preference p, investment k and TFP shock z)
   (3 of the shocks are almost perfectly observed)

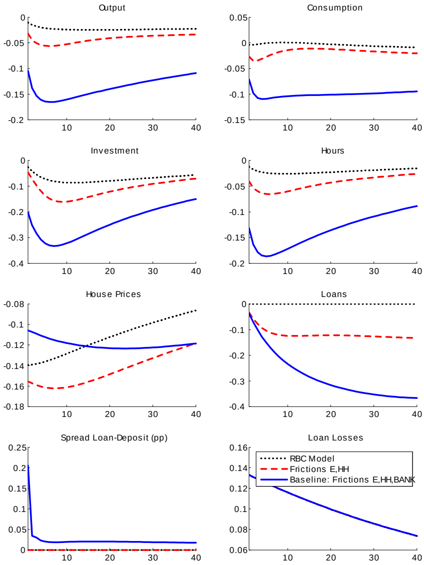


Consumption
Investment
Loan Losses Entr.
Loan Losses, HH
Loans Entr.
Loans HH
House Prices
Technology
% dev. from trend
% of annual GDP
1990
1995
2000
2005
2010

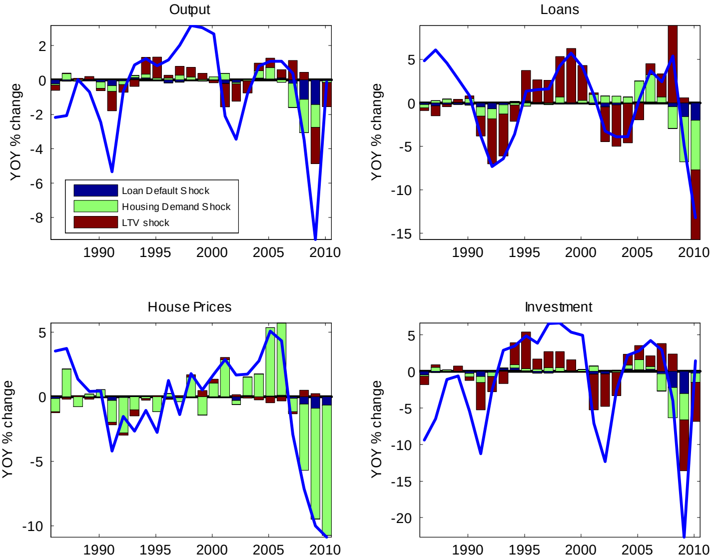
Remarks

1. I am using net charge-offs for banks from the data, and assuming that these charge-offs apply to the stock of mortgage and non–mortgage liabilities of HH and firms (which is larger than the stock of bank loans).
   Data: cumulative loan losses for comm. banks from 2007 to 2009 around $450bn. Banks "own" 1/3 of all debt instruments of households/firm: implied losses in the model are 3 times larger.
2. I am assuming that lenders cannot offset future expected charge–offs with higher interest rates

Financial Shock (blue: baseline model, red: model without banks)



Historical Decomposition



Historical Decomposition

Table 3: Historical Decomposition

Contribution to Output growth of	**2007**	**2008**	**2009**	**2010**	**2007-2**
Default shocks	*-0.2*	*-1.1*	*-1.4*	*-0.1*	[illegible]
Housing Demand shock	*-1.4*	*-1.9*	*-1.3*	*-0.1*	[illegible]
LTV shocks	*1.1*	*0.4*	*-2.1*	*-1.4*	[illegible]
Preference shock	3.3	0.3	-5.2	3.1	
TFP shocks	-2.5	-1.2	0.7	-1.6	
All shocks (data)	**0.4**	**-3.5**	**-9.4**	**-0.1**	**-1**

Contribution to Investment growth of	**2007**	**2008**	**2009**	**2010**	**2007-2**
Default shocks	*-0.3*	*-2.2*	*-3.0*	*-0.2*	[illegible]
Housing Demand shock	*-2.3*	*-4.1*	*-3.6*	*-1.2*	[illegible]
LTV shocks	*3.8*	*2.4*	*-7.0*	*-5.3*	[illegible]
Preference shock	3.1	-0.7	-6.7	6.6	
TFP shocks	-1.3	0.6	-2.9	1.8	
All shocks (data)	**2.9**	**-4.0**	**-23.2**	**1.6**	**-2**

Note: Contribution of each estimated shock to annual growth in Output (sum of consumption
residential fixed investment) and Investment.

The Timing of the Shocks

1. First stage of financial crisis 2007-2008: Housing Demand Shock drives drop in output
2. Second Stage 2008-2009: Redistribution Shock
3. Third Stage 2009-2010: LTV Shock
   Estimation tells a story in search of a unifying model (and perhaps one single shock): the decline in housing prices causes defaults which in turn cause tighter credit standard.

A DSGE Model of the Housing Market

- Two questions:

1. What is the nature of the shocks hitting the housing market?
2. How big are spillovers from the housing market to the wider economy?

- To answer them we build and estimate a quantitative model with:
  - nominal rigidities and monetary policy;
  - multi-sector structure with housing;
  - financing frictions on the household side.

Housing and the Macroeconomy

	Household balance sheet, 2008 billion $	
A	**Assets**	**67,13**
B	Real Estate (Owner-Occupied Homes)	20,39
C	Residential Real Estate of Noncorporate Business (Rented Homes)	4,964
D	Other Tangible Assets	4,779
E	Financial Assets less Residential Real Estate of Noncorp. Business	36,99
F	**Liabilities**	**14,21**
H	**Household net worth**	**52,91**
	Housing wealth	25,362
	Non housing wealth	27,555

Table 1.1: Composition of Household Wealth in the United St

Housing and the Macroeconomy

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Table 1.1: Composition of Household Wealth in the United St

Model

- Production
  - $Y-$sector produces C, IK, intermediate goods (using K and N)
  - $IH-$sector produces new homes (using K, N, land and interm. goods)
  - Different trend progress across sectors (C, IK, IH)
- Households
  - Patients work, consume, buy homes, rent capital and land to firms and lend to impatient households
  - Impatients/Credit Constrained work, consume, buy homes and borrow against value of home
    (We set up preferences in a way that borrowing constraint is binding)
- Sticky prices in the non-housing sector, sticky wages in both sectors
- Real rigidities: habits in C, imperfect labor mobility, K adjustment costs, variabile K utilization

Firms

- Firms produce :

$$Y_t = \left(A_{ct} N_{ct}\right)^{1-\mu_c} \left(z_{ct} k_{ct-1}\right)^{\mu_c}$$
$$IH_t = \left(A_{ht} N_{ht}\right)^{1-\mu_h-\mu_b-\mu_l} \left(z_{ht} k_{ht-1}\right)^{\mu_h} k_{bt}^{\mu_b} l_{t-1}^{\mu_l}.$$

- Y_t : non-housing, sticky price sector,
  IH_t flex price sector
- Two types of households/workers of measure 1
  α : wage share of unconstrained households (lenders)
  $1-\alpha$: wage share of constrained households (borrowers)

$$N_c = n_c^{\alpha} n_c'^{1-\alpha}, N_h = n_h^{\alpha} n_h'^{1-\alpha}$$

Lenders

$$\max E_0 \sum_{t=0}^{\infty} (\beta G_C)^t \, z_t \left(\log \widetilde{c}_t + j_t \log h_t - \tau_t g\left(n_{ct}, n_{ht}\right) \right)$$

- subject to budget constraint:

$$c_t + \frac{k_{ct}}{A_{kt}} + k_{ht} + q_t \left(h_t - (1 - \delta_h) h_{t-1} \right) + b'_t$$

$$= \widetilde{R}_{ct} k_{ct-1} + \widetilde{R}_{ht} k_{ht-1} + R_{lt} l_{t-1} + Div_t + wage'_t + \frac{R_{t-1} b'_{t-1}}{\pi_t}$$

Borrowers

- Discount future more heavily ($\beta' < \beta$)

$$\max E_0 \sum_{t=0}^{\infty} \left(\beta' G_C\right)^t z_t \left(\log \tilde{c}'_t + j_t \log h'_t - \tau_t g\left(n'_{ct}, n'_{ht}\right)\right)$$

- subject to budget constraint

$$c'_t + q_t \left(h'_t - (1-\delta_h)\, h'_{t-1}\right) = wage'_t + b'_t - \frac{R_{t-1}}{\pi_t} b'_{t-1}$$

- and to borrowing constraint

$$b'_t \leq m E_t \left(q_{t+1} h'_t \pi_{t+1} / R_t\right)$$

m : loan-to-value ratio

Monetary Policy

$$R_t = (R_{t-1})^{r_R} \left(\pi_t^{r_\pi} \left(\frac{GDP_t}{G_C GDP_{t-1}} \right)^{r_Y} \overline{rr} \right)^{1-r_R} \frac{\mathsf{u}_{Rt}}{\mathsf{s}_t}$$

u_{Rt} : iid monetary policy shock

s_t : persistent inflation objective shock

In Guerrieri and Iacoviello (2013), we allow for ZLB

Shocks

- Stationary AR(1)
  z_t : preference (discount factor) shock
  j_t : housing demand shock (*or household technology shock*)
  τ_t : labor supply shock
  u_{Rt} : monetary shock (iid)
  s_t : inflation objective shock
  u_{pt} : markup/inflation shock (iid)
- Trend-stationary shocks

$$
\begin{aligned}
\ln A_{ct} &= t\ln(1+\gamma_{AC}) + \ln Z_{ct}, \quad \ln Z_{ct} = \rho_{AC}\ln Z_{ct-1} + u_{Ct} \\
\ln A_{ht} &= t\ln(1+\gamma_{AH}) + \ln Z_{ht}, \quad \ln Z_{ht} = \rho_{AH}\ln Z_{ht-1} + u_{Ht} \\
\ln A_{kt} &= t\ln(1+\gamma_{AK}) + \ln Z_{kt}, \quad \ln Z_{kt} = \rho_{AK}\ln Z_{kt-1} + u_{Kt}
\end{aligned}
$$

Model Workings

1. At a basic level, it works like an RBC model with sticky prices/wages in the $Y-$sector, like an RBC with flex prices/sticky wages in the $IH-$sector (added twist: IH sector produces durables)
2. Sector specific shocks or preference shocks can shift resources from one sector to the other
3. Housing collateral generates wealth effects on consumption

Trends

1. Log preferences and Cobb-Douglas yield balanced growth
2. C and qIH grow at the same rate over time.
3. IK can grow faster than C, thanks to A_K progress
4. IH can grow slower than C, if land is a limiting factor and A_H is slow
5. Long-run growth rates

$$
\begin{aligned}
\frac{\Delta C}{C} &= \gamma_{AC} + \frac{\mu_c}{1-\mu_c}\gamma_{AK} \\
\frac{\Delta IK}{IK} &= \gamma_{AC} + \frac{1}{1-\mu_c}\gamma_{AK} \\
\frac{\Delta IH}{IH} &= (\mu_h + \mu_b)\,\gamma_{AC} + \frac{\mu_c\,(\mu_h + \mu_b)}{1-\mu_c}\gamma_{AK} + (1 - \mu_h - \mu_l - \mu_b)\,\gamma_{AH} \\
\frac{\Delta q}{q} &= (1 - \mu_h - \mu_b)\,\gamma_{AC} + \frac{\mu_c\,(1 - \mu_h - \mu_b)}{1-\mu_c}\gamma_{AK} \\
&\quad - (1 - \mu_h - \mu_l - \mu_b)\,\gamma_{AH}
\end{aligned}
$$

2. Estimation

1. Use 10 time-series (1965Q1-2006Q4) for US
   per capita C, IH, IK, real housing price q
   R, π, sectoral hours N_c and N_h, sectoral wages Δw_c and Δw_h
2. Some parameters calibrated to match steady state ratios
   $\beta = 0.9925$, $\beta' = 0.97$, $m = 0.85$
   $Y = N_c^{0.65} k_c^{0.35}$, $IH = N_h^{0.70} k_h^{0.10} k_b^{0.10} l^{0.10}$
   Targets: $(K + qH)/GDP = 3.2$, $(qH)/GDP = 1.35$,
   $(\delta_h qH)/GDP = 0.06$
3. Other parameters (including degree of financing frictions) estimated by Bayesian techniques

3. Results

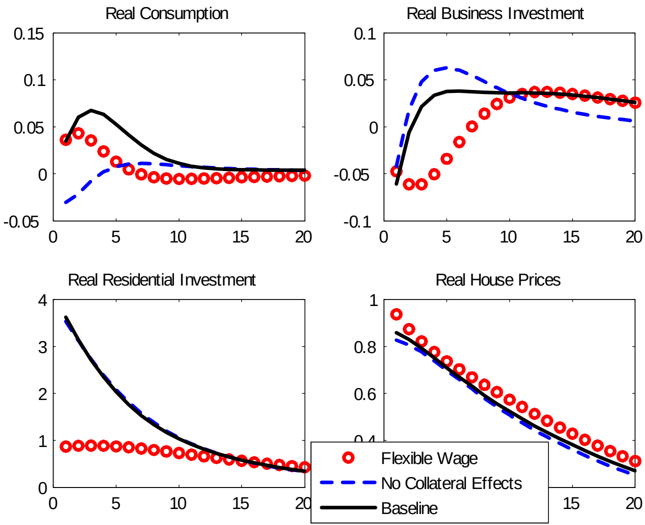
Prior and Posterior Parameters

1. Slow rate of technological progress in housing construction $(\gamma_{AC} = 0.32\%,\ \gamma_{AH} = 0.08\%)$
2. Wage share of credit constrained households $1 - \alpha =$21 percent
3. High price rigidity $(\theta_{\pi} = 0.83)$ and indexation $(\iota_{\pi} = 0.71)$
   High wage rigidity $(\theta_{wc} = 0.81, \theta_{wh} = 0.91)$, low wage indexation $(\iota_{wc} = 0.07, \iota_{wh} = 0.42)$
4. Taylor rule: $R_t = 0.61R_{t-1} + 0.39\left[1.38\pi_t + 0.51\left(gdp_t - gdp_{t-1}\right)\right]$

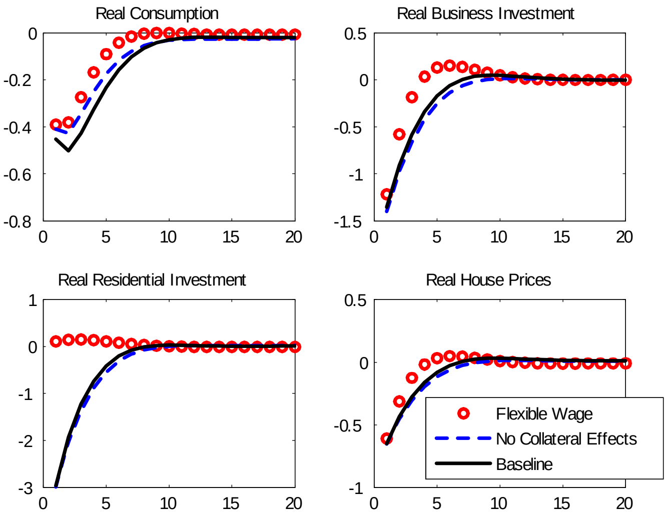
Variance Decomposition

Housing demand shocks and housing technology shocks account for one quarter each of the volatility of residential investment and house prices. Monetary shocks account for about 20 percent

Impulse Responses, Housing Preference Shocks



Impulse Responses, Monetary Shocks



Role of Monetary Shocks

1. Sensitivity of residential investment to monetary shocks larger than that of business investment, in line with VAR evidence
2. Key reason: wage stickiness
   If IH sector were flex wage, flex price, it would not contract after contractionary policy (BHK 2007)
3. Model elasticity of house prices to a monetary shocks of similar magnitude to what is found in VAR studies

Our two original questions, revisited.

1. What drives the housing market? Focus on recent period.
2. How big are the spillovers? Focus on pre and post 1980's

Drivers of Housing Market

Focus on 2000-2006:

Period		% q	Technology	Monetary Pol.
1998:I	2005:I	14.1	5.9	2.1
2005:II	2006:IV	-0.3	-0.2	-2.7
		% IH		
1998:I	2005:I	22.2	-4.1	9.8
2005:II	2006:IV	-15.5	-4.3	-11.4

Comparison with 1976-1985 period: monetary policy has played a larger role here.

Size of Spillovers

- Most of the spillovers are through the effect on consumption.
  For given LTV m, they are a function of α.
  Regression based on artificial data generated by the model

$$\begin{aligned} \Delta \log C_t &= 0.0041 + 0.123 \Delta \log HW_{t-1} \text{ if } \alpha = 0.79 \\ \Delta \log C_t &= 0.0041 + 0.099 \Delta \log HW_{t-1} \text{ if } \alpha = 1 \end{aligned}$$

- To better measure spillovers in sample, we re-estimate the model across subsamples (1965-1982, 1989-2006).
  First period: fix $m = 0.775$, $1 - \widehat{\alpha} = 0.33$
  Second period: fix $m = 0.925$, $1 - \widehat{\alpha} = 0.21$
- Two implications
  Monetary policy more "powerful" in the second period
  Housing shocks have larger spillover effects on consumption in the second period